The slides to the talk (and original paper) are posted on the conference web page. Here are some supplements that Adrian did at the board.

**Relationship with Good’s classical construction**

It is an amusing exercise to show that if

\[
\phi_v \left( \begin{array}{c} 1 \\ x \\ 1 \end{array} \right)
\]

depends only on \(|x|_v\) then

\[
\text{Pe}(g) = \sum_{N_Q \backslash G_Q} \phi(\gamma g) = \sum_{\text{SL}_2(\mathbb{Z})} \phi(\gamma g).
\]

To prove this, we use this fact, a proof of which may be found in the book of Iwaniec.

\[
M_Q \backslash G_Q = \bigcup_{r \in \mathbb{Q}} \quad \left( \begin{array}{c} 1 \\ r \\ 1 \end{array} \right) \text{SL}_2(\mathbb{Z})
\]

\[
G_\infty \cdot \text{SL}_2(\mathbb{Z})
\]

\[
\prod_{v \mid \infty} G_v \prod_{v < \infty} K_v^{\text{max}}.
\]

Strong approximation (Borel) the number of double cosets in \(G_\infty \backslash G_A / G_k\) equals the class number. So if \(k = \mathbb{Q}\) we have \(G_A = G_\infty G_k = G_k G_\infty\). We can write \(g = \gamma g_0\) where \(\gamma, g_0 \in G_k\). Now if

\[
\gamma_g = \gamma = \left( \begin{array}{c} 1 \\ r \\ 1 \end{array} \right) \delta, \quad \delta \in \text{SL}(2, \mathbb{Z}),
\]

our assumptions are

\[
\phi_v(g) = \begin{cases} x(\cdot) & \text{if } g \in M_vK_v \\ 0 & \text{if } r = 0. \end{cases}
\]

so we can discard this contribution unless \(r = 0\).