Adrian Diaconu, 7/13

The slides to the talk (and original paper) are posted on the conference web page. Here are some supplements that Adrian did at the board.

Relationship with Good's classical construction

It is an amusing exercise to show that if

$$\phi_v\begin{pmatrix} 1 & x \\ & 1 \end{pmatrix}$$

depends only on $|x|_v$ then

$$\mathrm{Pe}(g) = \sum_{N_{\mathbb{Q}}\backslash G_{\mathbb{Q}}} \phi(\gamma g) = \sum_{\mathrm{SL}_2(\mathbb{Z})} \phi(\gamma g).$$

To prove this, we use this fact, a proof of which may be found in the book of Iwaniec.

$$M_{\mathbb{Q}}\backslash G_{\mathbb{Q}} = \bigcup_{\substack{r\in\mathbb{Q} \\ 0\leqslant r<1}} \begin{pmatrix} 1 & r \\ & 1 \end{pmatrix}\mathrm{SL}_2(\mathbb{Z})$$

$$G_A^{\infty} = \prod_{v|\infty} G_{\infty} \prod_{v<\infty} K_v^{\max}.$$

Strong approximation (Borel) the number of double cosets in $G_A^{\infty}\backslash G_A/G_k$ equals the class number. So if $k=\mathbb{Q}$ we have $G_A = G_A^{\infty}G_k = G_kG_A^{\infty}$. We can write $g = \gamma_g g_0$ where $\gamma_g \in G_k$ and $g_0 \in G_A^{\infty}$. Now if

$$\gamma_g = \gamma = \begin{pmatrix} 1 & r \\ & 1 \end{pmatrix}\delta, \qquad \delta\in\mathrm{SL}(2,\mathbb{Z}),$$

our assumptions are

$$\phi_\nu(g) = \begin{cases} \chi() & \\ 0 & \text{if } g\in M_\nu K_\nu \end{cases}$$

so we can discard this contribution unless $r=0$.