Correction to 'Toeplitz Minors'

Daniel Bump and Persi Diaconis

July 14, 2003

Barry Simon pointed out a point requiring clarification in the proof of Theorem 4. After the displayed formula

$$e^{nc_0}\int_{U(n)}\prod_{k=1}^{\infty}\sum_{a_k=0}^{\infty}\frac{(c_k\,\mathrm{tr}(g^k))^{\alpha_k}}{\alpha_k!}\sum_{a_k=0}^{\infty}\frac{(c_{-k}\overline{\mathrm{tr}(g^k)})^{\beta_k}}{\beta_k!}dg,$$

it is asserted that "expanding this and invoking (1.11) and (1.12), the only terms which survive have $\alpha_k=\beta_k$."

This is an oversimplification. It is true that if either $\sum k\alpha_k\leqslant n$ or $\sum k\beta_k\leqslant n$ then the term is killed by (1.11). However if $\sum k\alpha_k>n$ and $\sum k\beta_k>n$ all we can immediately assert is that $\sum k\alpha_k=\sum k\beta_k$. (For this, integration over the center is sufficient to kill the term if $\sum k\alpha_k\neq\sum k\beta_k$.)

To correct this point, we will not use (1.12) but just the fact that the characteristic map $R_m\longrightarrow\Lambda_m$ is a subisometry (first displayed formula after (1.8)).

Expanding and collecting terms with $\sum k\alpha_k=\sum k\beta_k=m$, we obtain

$$e^{nc_0}\sum_{m=0}^{\infty}C(m,n),$$

where

$$C(m,n)=\left\langle\sum_{\substack{(\alpha_k)\\ \sum k\alpha_k=m}}\left(\prod_k\frac{c_k^{\alpha_k}}{\alpha_k!}\right)p_{\lambda_\alpha},\ \sum_{\substack{(\beta_k)\\ \sum k\beta_k=m}}\left(\prod_k\frac{\overline{c_{-k}}^{\beta_k}}{\beta_k!}\right)p_{\lambda_\beta}\right\rangle.$$

Here as in the paper λ_α is the partition with α_k parts equal to k. When $m\leqslant n$, we may simply expand the inner product and use (1.11) to simplify the result as in the paper. When $m\leqslant n$ we can discard the terms with $\alpha_k\neq\beta_k$ and write

$$C(m,n)=\sum_{\substack{(\alpha_k)\\ \sum k\alpha_k=m}}\left(\prod_k\frac{c_k^{\alpha_k}}{\alpha_k!}\right)\left(\prod_k\frac{c_{-k}{}^{\alpha_k}}{\alpha_k!}\right)\langle p_{\lambda_\alpha},p_{\lambda_\alpha}\rangle=C(m)$$

where

$$C(m)=\sum_{\substack{(\alpha_k)\\ \sum k\alpha_k=m}}\prod_k\frac{(kc_kc_{-k})^{\alpha_k}}{\alpha_k!}.$$

This much is as in the paper. If $n > m$ we will show that

$$|C(m,n)| \leqslant |D(m)|,$$

where the series

$$\sum_m |D(m)| < \infty.$$

This is clearly sufficient for the purposes of the theorem.

First consider the case where $c_k = \overline{c_{-k}}$. In this case, we can take $D(m) = C(m)$.

$$|C(m,n)| = \left\| \sum_{\substack{(\alpha_k) \\ \sum k\alpha_k = m}} \left(\prod_k \frac{c_k^{\alpha_k}}{\alpha_k!} \right) p_{\lambda_\alpha} \right\|^2$$

which is dominated by the corresponding inner product in R_m because the characteristic map is a subisometry. Thus (with $c_k = \overline{c_{-k}}$)

$$|C(m,n)| \leqslant \sum_{\substack{(\alpha_k) \\ \sum k\alpha_k = m}} \sum_{\substack{(\beta_k) \\ \sum k\beta_k = m}} \left(\prod_k \frac{c_k^{\alpha_k}}{\alpha_k!} \right) \left(\prod_k \frac{\bar{c}_k^{\beta_k}}{\beta_k!} \right) \langle z_{\lambda_\alpha} f_{\lambda_\alpha}, z_{\lambda_\beta} f_{\lambda_\beta} \rangle$$

where now the inner product is in R_m. The right-hand side equals $C(m)$ as before. Note that because this is an inner product on R_m the f_{λ_α} are orthogonal, even though (if $m > n$) their images under the characteristic map are not orthogonal.

If c_k and $\overline{c_{-k}}$ are not assumed equal, we may use the Cauchy-Schwarz inequality and write

$$|C(m,n)| \leqslant \left\| \sum_{\substack{(\alpha_k) \\ \sum k\alpha_k = m}} \left(\prod_k \frac{c_k^{\alpha_k}}{\alpha_k!} \right) p_{\lambda_\alpha} \right\| \cdot \left\| \sum_{\substack{(\alpha_k) \\ \sum k\alpha_k = m}} \left(\prod_k \frac{c_{-k}^{\alpha_k}}{\alpha_k!} \right) p_{\lambda_\alpha} \right\|.$$

Each norm is dominated by the corresponding norm in R_k and proceeding as before, we obtain the same bound with c_k replaced by $\max(|c_k|, |c_{-k}|)$.