OPEN AND CLOSED MODELS.

TOKUYAMA MODELS: POSSIBLE SPINS WERE $\oplus$ AND $\ominus$. PARTITION FUNCTIONS:

$$Z(\mathfrak{S}_\lambda^q) = \prod_{i>j} (1 - q z_i/z_j)\,\Delta_\lambda(z)$$

$q=0$ $\quad Z(\mathfrak{S}_\lambda) = \Delta_\lambda(z)$

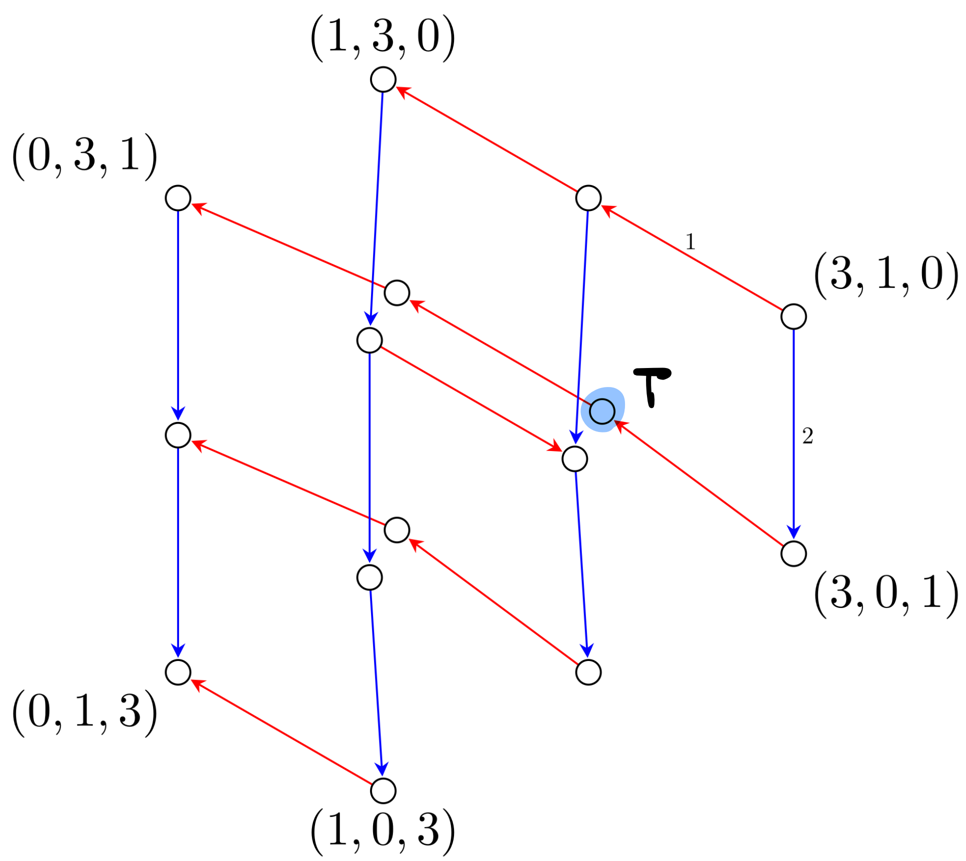
INFORMATION COMING FROM YBE: PARTITION FUNCTION IS SYMMETRIC.

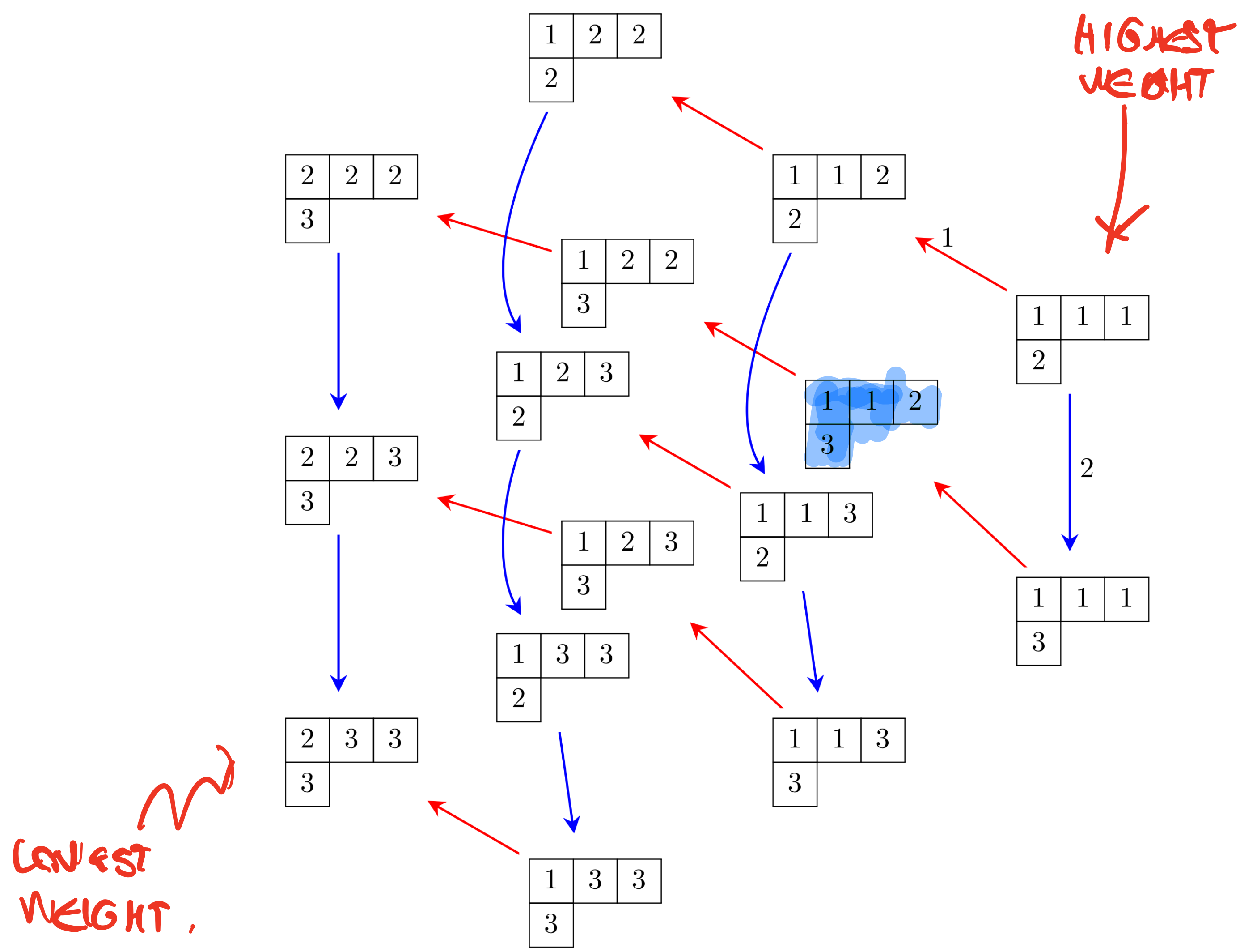
$q=0$ STATES ARE IN BIJECTION WITH

$\mathcal{B}_\lambda$ = CRYSTAL OF SSYT OF SHAPE λ.

STATE $\rightsquigarrow$ GELFAND TSETLIN PATTERN $\leftrightarrow$ SSYT $\xrightarrow[\text{INVOLUTION ON TABLEAUX}]{}$ SSYT.

THE SCHUTZENBERGER INVOLUTION ON TABLEAUX: $\sigma(T)$ REPLACES T WITH WT μ BY ANOTHER TABLEAU OF WT $w_0\mu$


(1, 3, 0)
(0, 3, 1)
(3, 1, 0)
(3, 0, 1)
(0, 1, 3)
(1, 0, 3)
1
2
T


HIGHEST WEIGHT
1
2
LOWEST WEIGHT.

$\mathcal{B}_\lambda$ AS A GRAPH HAS AN AUTOMORPHISM INTERCHANGING HIGHEST WEIGHT ELT

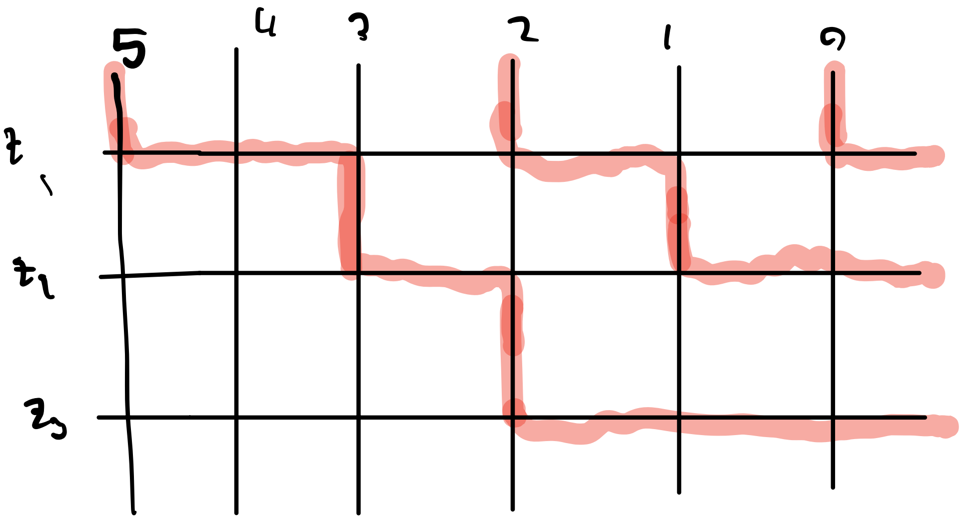
$$\begin{array}{|c|c|c|}\hline 1&1&1\\\hline 2\\\cline{1-1}\end{array} \mapsto \begin{array}{|c|c|c|}\hline 2&3&3\\\hline 3\\\cline{1-1}\end{array}$$

σ REVERSES $i \mapsto n-i$

$1 \mapsto 2$ FOR $GL(3)$

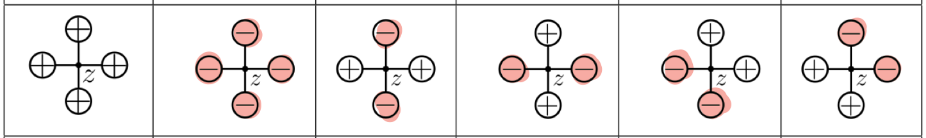
$$\sigma(f_i(x)) = e_{n-i}(\sigma(x))$$
$$\sigma(e_i(x)) = f_{n-i}(\sigma(x)).$$

LUSZTIG GENERALIZED TO ALL CARTAN TYPES.



$z_1^3 z_2^2 z_3^2$

$\lambda = (3,1,0)$ $\lambda + \rho = (5,2,0)$

a_1	a_2	b_1	b_2	c_1	c_2
					
1	z	$-q$	z	$z(1-q)$	1

$q=0$

$$\mathrm{GTP}(\text{STATE}) = \left(\begin{array}{ccccc} 5 & & 2 & & 0 \\ & 3 & & 1 & \\ & & 2 & & \end{array}\right) \qquad \text{SUBTRACT} \quad \begin{array}{ccccc} 2 & & 1 & & 0 \\ & 1 & & 0 & \\ & & 0 & & \end{array}$$

$$\mathrm{GTP}^0(\text{STATE}) = \begin{array}{ccccc} 3 & & 1 & & 0 \\ & 2 & & 1 & \\ & & 2 & & \end{array}$$

PRODUCE A TABLEAU FROM GTP^0

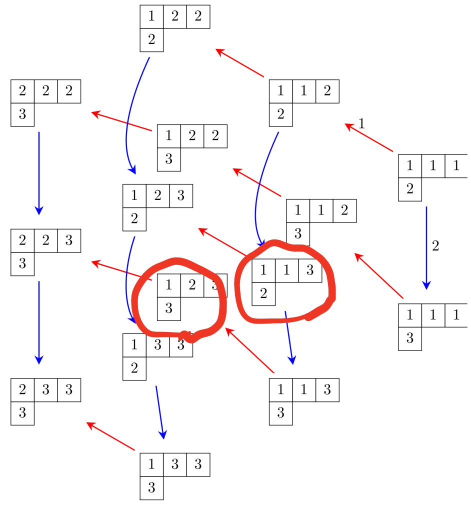
$$\begin{array}{|c|c|c|} \hline 1 & 1 & 3 \\ \hline 2 & \multicolumn{2}{c}{} \\ \cline{1-1} \end{array} \xrightarrow[\text{REMOVE } 3]{} \begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & \multicolumn{1}{c}{} \\ \cline{1-1} \end{array} \xrightarrow[\text{REMOVE } 2]{} \begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array}$$

$(3,1,0) \qquad\qquad (2,1) \qquad\qquad (2)$

ONE MORE STEP: APPLY σ

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 3 & \multicolumn{2}{c}{} \\ \cline{1-1} \end{array}$$

$$f_2 f_1 \begin{pmatrix} 1 & 1 & 1 \\ 2 & & \end{pmatrix}$$



HIGHEST

LOWEST.

$(2,1,1)$

$(1,1,2)$

$$\mathrm{wt}\left(\begin{array}{|c|c|c|} \hline 1 & 2 & 3 \\ \hline 3 & \multicolumn{2}{c}{} \\ \cline{1-1} \end{array}\right) = (1,1,2).$$

CLAIM: IF $\Delta \leftrightarrow T$

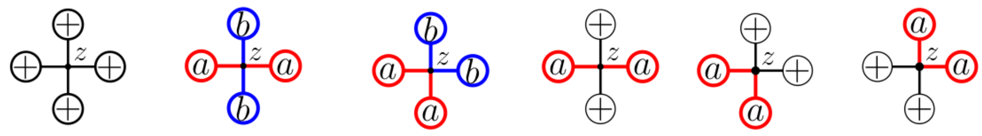
THEN $$\beta(\Delta) = z^{\rho} \cdot z^{\mathrm{wt}(T)}$$

$$\beta(\Delta) = z_1^3 z_2^2 z_3^2 = z^{\rho} \cdot z_1 z_2 z_3^2$$

THE CLOSED MODELS ARE COLORED VARIANTS WHOSE PARTITION FUNCTIONS ARE DEMAZURE CRYSTALS.

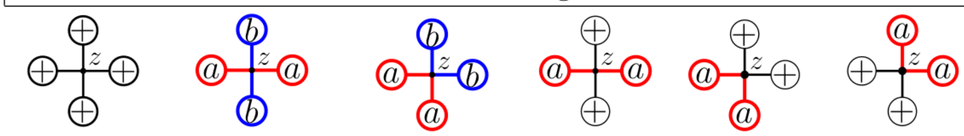
Y. YANG, BBBG, BUCIUMAS-SCRIMSHAW.
OPEN MODELS.

Open T-weights					



1	$z \quad a \geqslant b$ $0 \quad a < b$	$0 \quad a > b$ $z \quad a < b$	z	z	1

Closed T-weights					



1	$z \quad a \geqslant b$ $0 \quad a < b$	$z \quad a > b$ $0 \quad a < b$	z	z	1

LET n COLORS BE GIVEN.

SPINS ARE $+, c_1, \dots, c_n$.

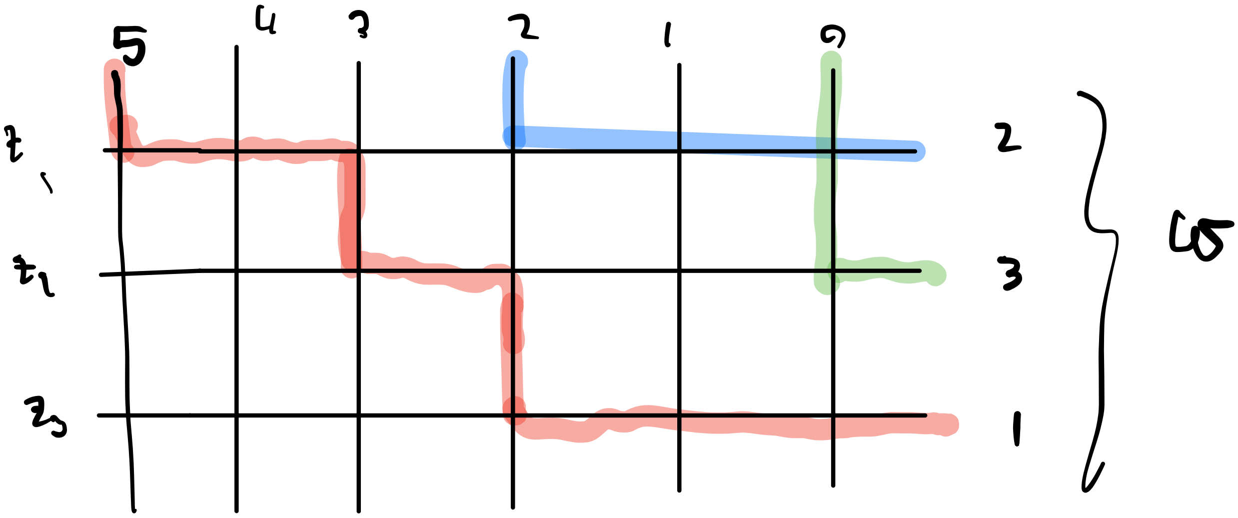
MODIFY TOKUYAMA MODELS:

TOP BOUNDARY EDGES HAVE A COLOR c_i AT $\lambda_i + n - i$ COLUMN

$$c_1 > c_2 > \dots > c_n$$

AND ON THE RIGHT SIDE USE SOME ORDER DEPENDING ON A PERMUTATION.

c_1: RED $\quad c_2$: BLUE $\quad c_3$: GREEN.



ON FRIDAY I WILL SHOW USE YBE

$$Z(S_\omega) = z^\rho \partial^\circ_\omega z^\lambda \qquad \text{OPEN}$$

$$z^\rho \partial_w z^\lambda \qquad \text{CLOSED.}$$

DEEPER FACT: SET OF STATES IS NATURALLY IN BIJECTION WITH

DEMAZURE CRYSTAL (CLOSED)

DEMAZURE ATOM (OPEN)

$$\mathcal{B}_\lambda(w) = \bigcup_{y \leq w} \mathcal{B}^0_\lambda(y)$$

DEMAZURE CRYSTAL

BRUHAT

DEMAZURE ATOMS

$$\mathcal{B}_\lambda = \bigcup \text{ ATOMS}.$$