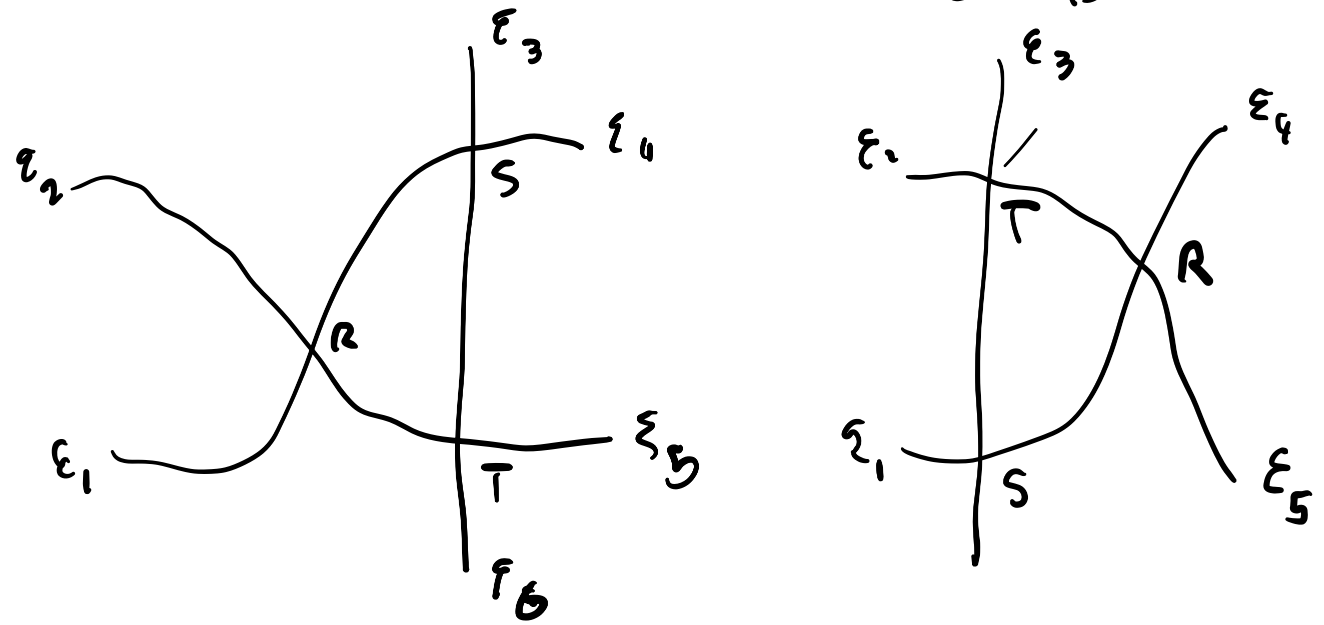
PARAMETRIZED YANG-BAXTER EQUATIONS.

(1) EQUIVALENCE OF TWO SYSTEMS

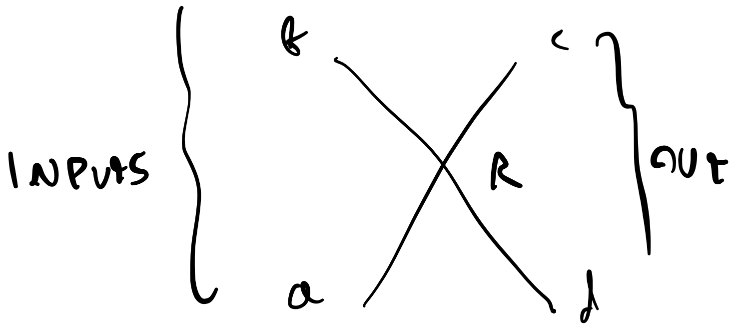


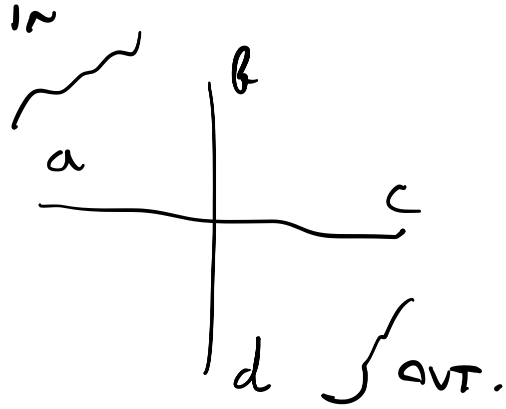
MOVING R ACROSS THE S, T VERTICES SWITCHES THEM USED IN TRAIN ARGUMENT.

(2) "LINEAR ALGEBRA STATEMENT"

$\varepsilon_i \in \Sigma$ A SET OF ALLOWED SPINS.

IF V = FREE VECTOR SPACE ON Σ, WE CAN REGARD A VERTEX AS AN ENDOMORPHISM OF $V \otimes V$ AS FOLLOWS.





$a, b, c, d \in \Sigma$

$a \otimes b \in V \otimes V$

$d \otimes c$

$R : V \otimes V \to V \otimes V$ IS THE LINEAR TRANSFORMATION

$\beta\left({}_{a}^{b} X_{d}^{c} \right)$ = COEF IN $R(a \otimes b)$ OF $d \otimes c$

$$\langle d \otimes c \,|\, R \,|\, a \otimes b \rangle = \beta\left({}_{a}^{b} X_{d}^{c} \right)$$

$T : V \otimes V \to V \otimes V \qquad x \otimes y \to y \otimes x$

$\tau \circ R$ IS OFTEN CONSIDERED THE R-MATRIX.

BOTH CONVENTIONS ARE USED.

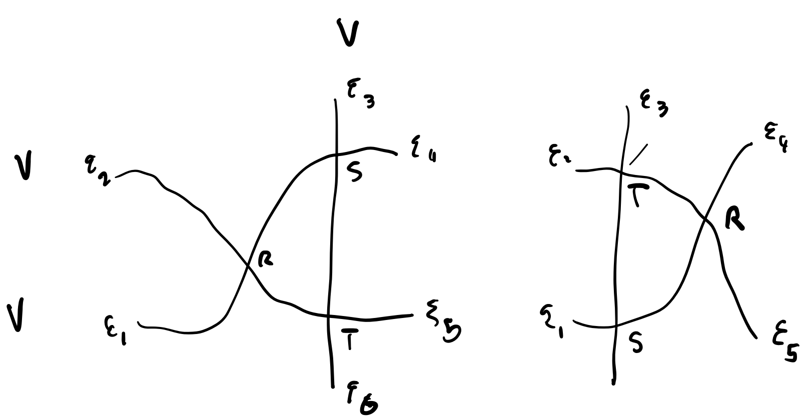
YANG BAXTER EQUATION IS AN IDENTITY IN $V \otimes V \otimes V$

$$(T \otimes I_V)(I_V \otimes S)(R \otimes I_V)$$

$$= (I_V \otimes R)(S \otimes I_V)(I_V \otimes T_V)$$

THIS IS EQUIVALENT TO OTHER FORMULATION.

REF; SECTION 6 OF CH. I.



$[[R, S, T]]$ $R, S, T \in \mathrm{END}(V \otimes V)$

$= (T \otimes I)(I \otimes S)(R \otimes I)$
$- (I \otimes R)(S \otimes I)(I \otimes T)$

"YANG BAXTER COMMUTATOR"

$[[R, S, T]] = 0$ SUCCINCT YBE.

PARAMETRIZED YBE

LET Γ BE A GROUP

USUALLY: $\mathbb{C}^\times$, $\mathbb{C}$, ELLIPTIC CURVE

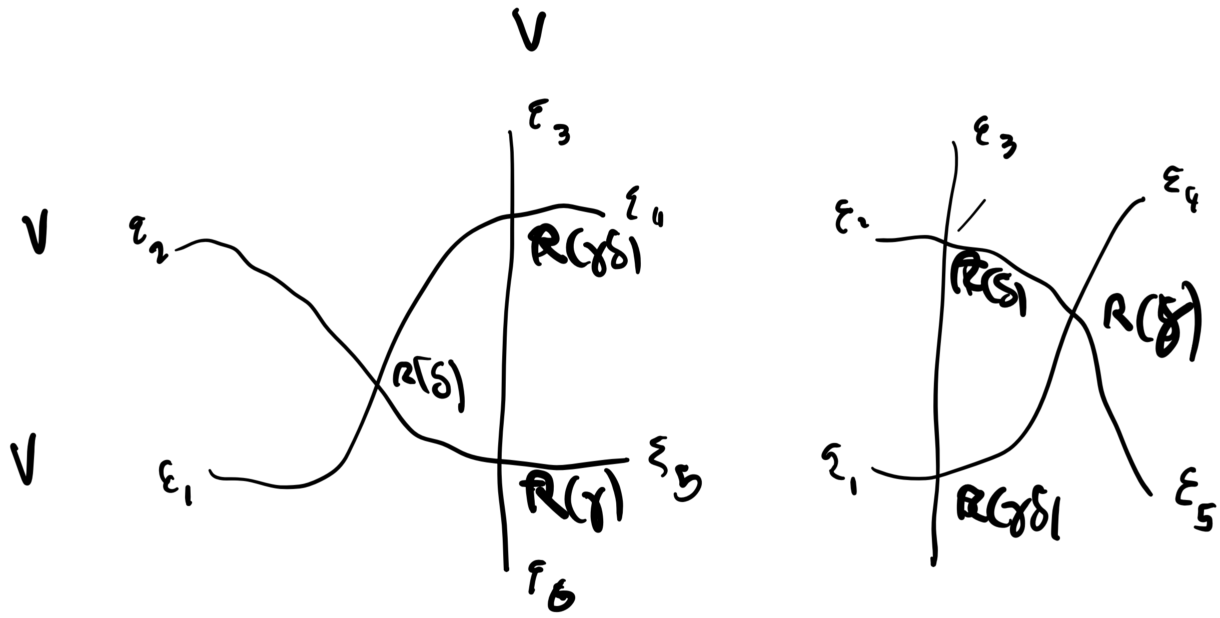
ONE EXOTIC EXAMPLE: $GL(2) \times GL(1)$

"FREE-FERMIONIC SIX VERTEX MODEL" (TOKUYAMA).

V SOME VS.

$R : \Gamma \to \mathrm{END}(V \otimes V)$ SOME MAP

$$[[R(\gamma), R(\gamma\delta), R(\delta)]] = 0$$



PARAMETRIZED YBE

VARIANT: HOMOGENEOUS PARAMETRIZED YBE

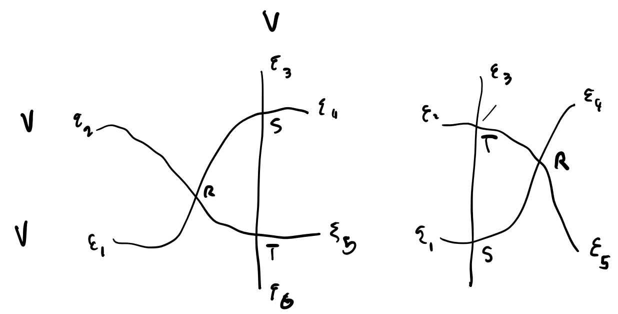
$z_1, z_2 \in \mathbb{C}^\times$

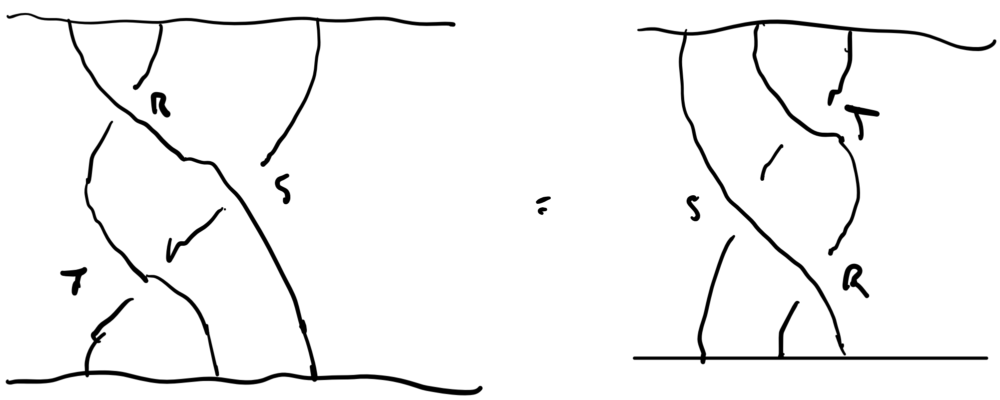
$$R(\gamma_1, \gamma_2)$$

$$[[R(\gamma_1, \gamma_2), R(\gamma_1, \gamma_3), R(\gamma_2, \gamma_3)]] = 0$$

MANY EXAMPLES.

NEXT LECTURE WE WILL CONNECT HOMOGENEOUS PARAMETRIZED YBE TO DEMAZURE OPS EXPLAINED IN CHAPTER 6 OF THE TEXT.





YBE ⇒ BRAID RELATIONS FOR DEMAZURE OPERATORS.