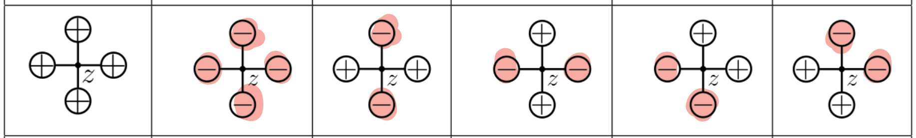
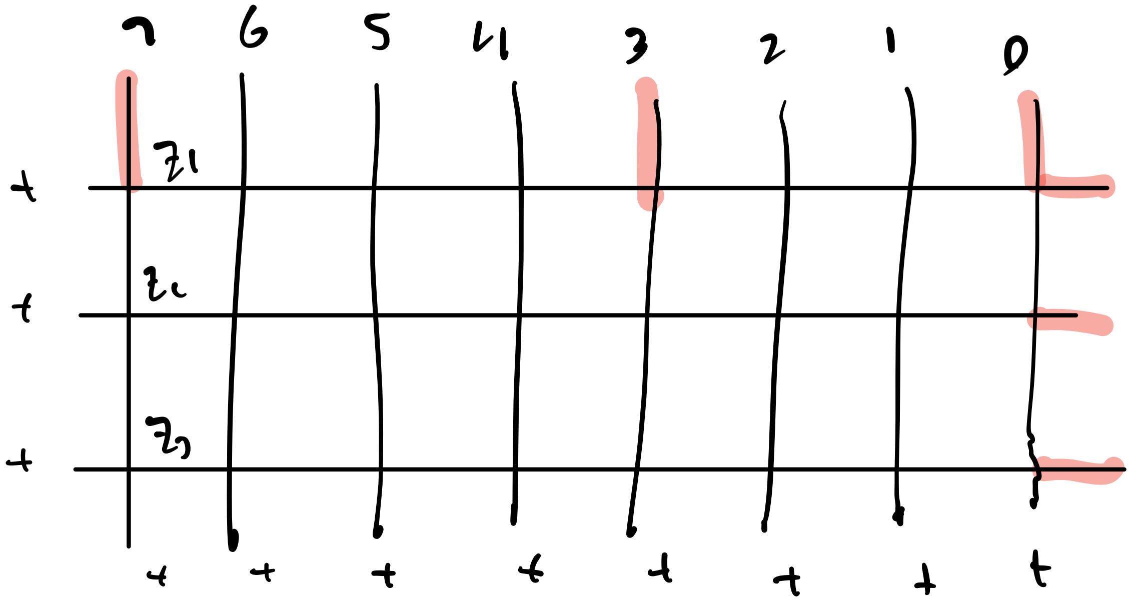
$\lambda = (\lambda_1, \cdots, \lambda_n)$

$\rho = (n-1, n-2, \cdots, 0)$

a_1	a_2	b_1	b_2	c_1	c_2
					
1	z	$-q$	z	$z(1-q)$	1

$z = z_i$ USE THESE WEIGHTS IN i-TH ROW



$\ominus$ ON TOP BOUNDARY COLUMNS $\lambda_i + n - i$

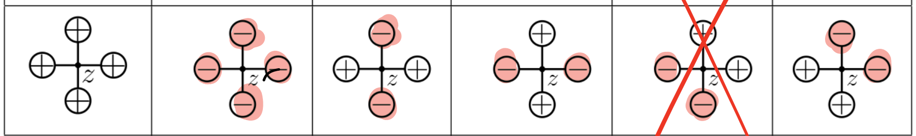
$\lambda = (5, 2, 0)$ $\qquad \lambda + \rho = (7, 3, 0)$

$n = 3$

$$Z(S_\lambda(z;q)) = \frac{S_\lambda(z_1, \cdots, z_n)}{\prod_{i<j}(z_i - q z_j)}$$

WHERE S_λ IS A SYMMETRIC POLY (NOT INVOLVING q).

$q = 1$ LEADS TO $n! = |W|$ STATES



a_1	a_2	b_1	b_2	~~c_1~~	c_2
1	z	$-q$	z	~~$z(1-q)$~~	1

$$S_\lambda(z) = \frac{\sum_w \cdots}{\prod_{i<j} z_i - z_j}$$

$$= \frac{\sum (-1)^{\ell(w)} w(z^{\lambda+\rho})}{\prod} = \Delta_\lambda(z_1, \dots, z_n)$$

USUAL SCHUR.

$q = 0$ THERE ARE NO b_1 PATTERNS.

STATES ARE IN BIJECTION WITH GELFAND-TSETLIN PATTERNS

AND SSYT (SEMI-STANDARD YOUNG TABLEAUX).

$\rightsquigarrow$ COMBINATORIAL DEF OF SCHUR FUNCTION.

A GELFAND TSETLIN PATTERN OF SIZE n AND SHAPE λ

ROWS INDEXED BY $0, 1, 2, \dots$

0-TH ROW IS λ

EACH ROW IS A PARTITION $(a_{i,1}, \dots, a_{i,n-i})$

ROWS INTERLEAVE

$$\lambda = (\lambda_1, \dots, \lambda_n)$$
$$\mu = (\mu_1, \dots, \mu_{n-1})$$

$$\lambda_1 \geq \mu_1 \geq \lambda_2 \geq \mu_2 \geq \dots \geq \lambda_n$$

$n = 3$

$$\begin{array}{ccccc} \lambda_1 & & \lambda_2 & & \lambda_3 \\ & a & & b & \\ & & c & & \end{array}$$

$$\lambda_1 \geq a \geq \lambda_2 \geq b \geq \lambda_3$$
$$a \geq c \geq b.$$

$GL(n) \rightsquigarrow GL(n-1)$ BRANCHING RULE.

$\lambda = (\lambda_1, \dots, \lambda_n)$ A PARTITION

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n$$

THERE IS A IRR REP'N OF $GL(n)$ OF HIGHEST WEIGHT λ.

THEOREM: $\dim \pi_\lambda^{GL(n)}$ = # OF GTP WITH TOP ROW λ.

FOLLOWS FROM BRANCHING RULE

$$GL(n-1) \rightsquigarrow GL(n)$$

$$g \rightsquigarrow \begin{pmatrix} g & \\ & 1 \end{pmatrix}$$

$$\pi_\lambda^{GL(n)}\Big|_{GL(n-1)} = \bigoplus_{\substack{\mu = (\mu_1, \dots, \mu_{n-1}) \\ \lambda, \mu \text{ INTERLEAVE.}}} \pi_\mu^{GL(n-1)}$$

GTP FOR λ READ OFF μ FROM SECOND ROW.

$$\#\text{ GTP OF SHAPE } \lambda \text{ SIZE } n = \sum_{\mu, \lambda \text{ INTERLEAVING}} \#\text{ GTP OF SHAPE } \mu.$$

BIJECTION GTP WITH TOP ROW λ AND SSYT OF SHAPE λ.

IF WE DISCARD BOXES IN A SSYT CONTAINING n OBTAIN A SSYT OF SHAPE μ SOME μ INTERLEAVING λ.

$$T : \begin{array}{|c|c|c|c|c|c|} \hline 1 & 1 & 2 & 3 & 3 & 5 \\ \hline 2 & 3 & 3 & 4 & \multicolumn{2}{c}{} \\ \cline{1-4} 5 & 5 & \multicolumn{4}{c}{} \\ \cline{1-2} \end{array}$$

$n = 5$

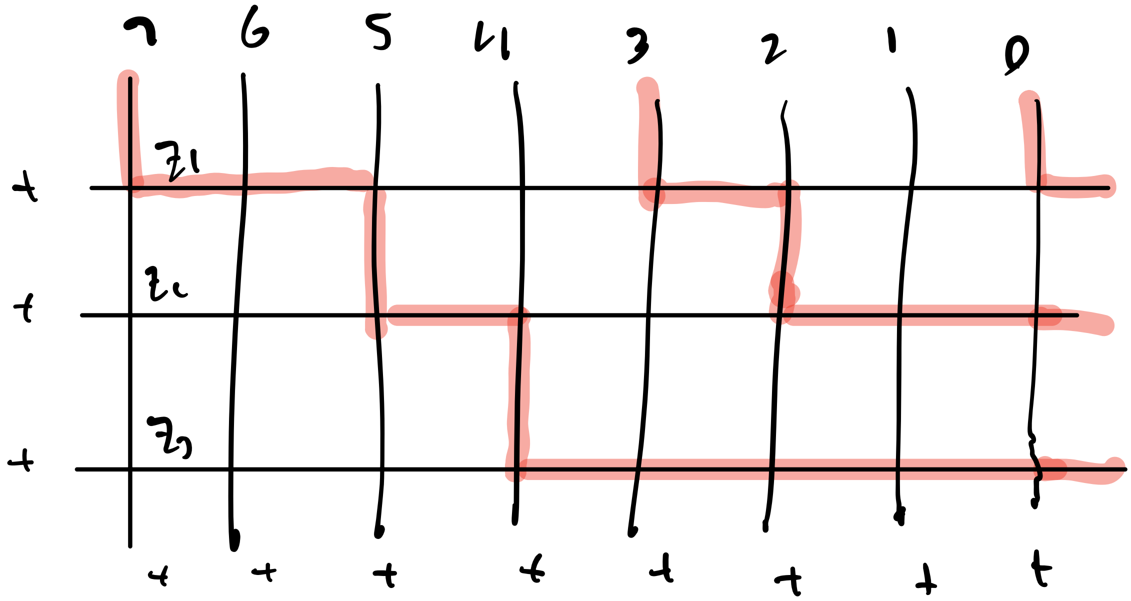
$\lambda = (6, 4, 2)$

$\mu = (5, 4)$

CONTINUE: ELIMINATE BOXES LABELED $n-1$ OBTAIN A SEQUENCE OF PARTITIONS

$$G : \begin{array}{ccccccccc} 6 & & 4 & & 2 & & 0 & & 0 \\ & 5 & & 4 & & 0 & & 0 & \\ & & 5 & & 3 & & 0 & & \\ & & & 3 & & 1 & & & \\ & & & & 2 & & & & \end{array}$$

GIVEN A STATE OF SIX VERTEX MODEL THE VERTICAL EDGES CONTAINING ⊖ SPINS GIVE A GTP.



$\lambda_i + n - i$

$= (\lambda + \rho)_i$

7 3 0

5 2

4

ENTRIES IN GTP TELL US WHICH COLS HAVE − SPINS.

$(a_{i,j})$ STRICT GTP OF SIZE n TOP ROW $\lambda+\rho$ $\longleftrightarrow$ $S_\lambda(z;q)$

$\cap$

GTP OF TOP ROW $\lambda+\rho$

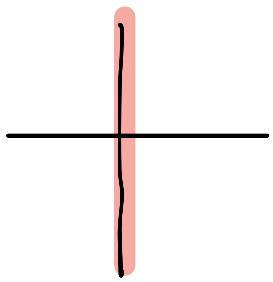
$\updownarrow$ BIJECTION

SSYT OF SHAPE $\lambda+\rho$

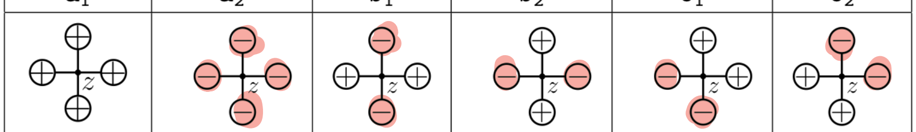
STRICT MEANS

$$a_{N,1} > a_{N,2} > a_{N,3} > \dots$$

IF $q = 0$



THIS CAN'T HAPPEN



a_1	a_2	b_1	b_2	c_1	c_2
1	z	$-q$	z	$z(1-q)$	1

1 z 0 z z 1

1 1 0 z 1 1 AGGARWAL.

WEIGHTS ARE CALLED FREE-FERMIONIC IF

$$a_1 a_2 + b_1 b_2 = c_1 c_2$$

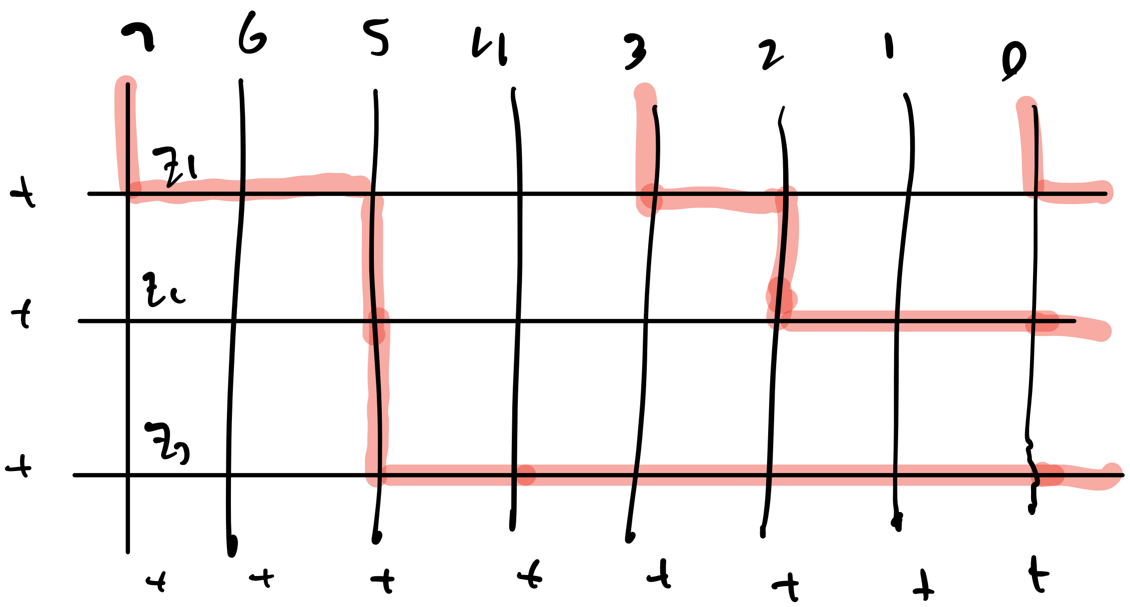
FOR FREE-FERMIONIC WEIGHTS THERE IS A "LARGER" YANG BAXTER EQUATION MANY INTERESTING EXAMPLES.

THE GTP IS LEFT STRICT

$$a_{i,j} \quad a_{i,j+1}$$

$$\downarrow$$

$$a_{i+1,j}$$



7 3 0

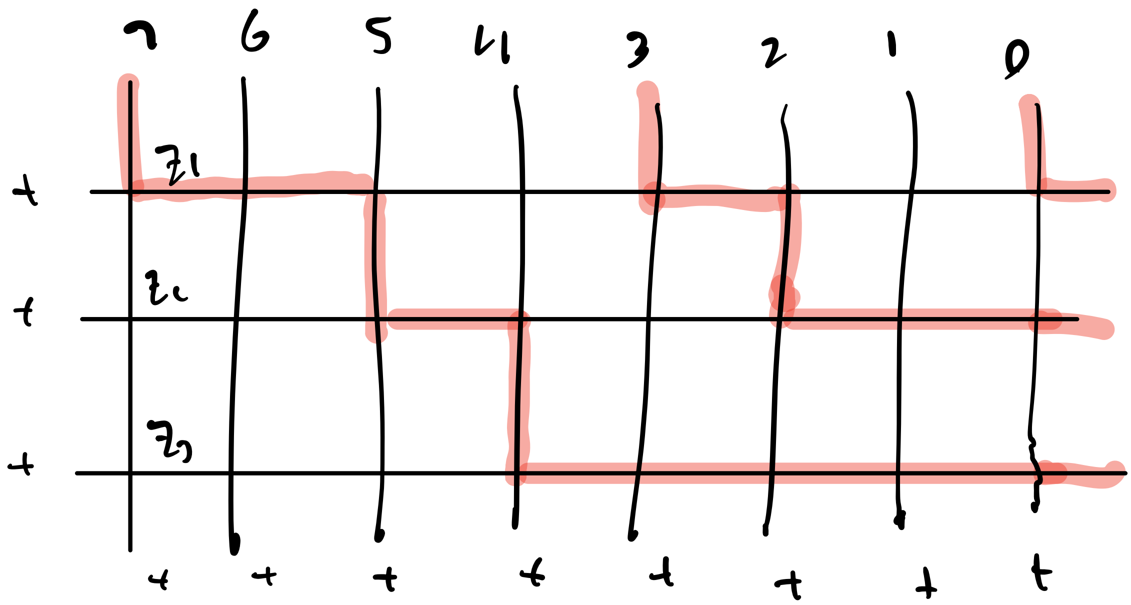
5 2

5

NOT LEFT STRICT.

LEFT STRICT MEANS WE CAN SUBTRACT

$$P = \begin{array}{ccccc} n & n-1 & n-2 & \cdots & 0 \\ & n-1 & n-2 & \cdots & 0 \\ & & n-2 & & \\ & & \ddots & ⋰ & \\ & & & 0 & \end{array}$$



$$\begin{array}{ccccc} 7 & & 3 & & 0 \\ & 5 & & 2 & \\ & & 4 & & \end{array} \quad - \quad \begin{array}{ccccc} 2 & & 1 & & 0 \\ & 1 & & 0 & \\ & & 0 & & \end{array}$$

$$GTP^{0}(\Delta) = GTP(\Delta) - \rho$$

$$\begin{array}{ccccc} 5 & & 2 & & 0 \\ & 4 & & 2 & \\ & & 4 & & \end{array}$$

USING THESE REDUCE GTP

STATES $q=0$ $\leftrightarrow$ GTP TOP ROW λ $\leftrightarrow$ SSYT OF SHAPE λ.

$$\beta(\Delta) = z^{\rho} \cdot z^{w_0\, wt(T)}$$

$$z\left(s_\lambda(z;q)\right) = \underbrace{\prod(z_i - \cancel{q} z_j)}_{zP} s_\lambda(z_1, \dots, z_n)$$

so $\quad s_\lambda(z) = \sum_{\text{SSYT}} z^{w_0\,wt(T)}$

$$(w_0 z)^{wt(T)}$$

BUT WE PROVED s_λ IS SYMMETRIC

so $\quad s_\lambda(z) = \sum z^{w_0\,wt(T)}$

$$= \sum z^{wt(T)} =$$

COMBINATORIAL DEF OF Δ_λ.