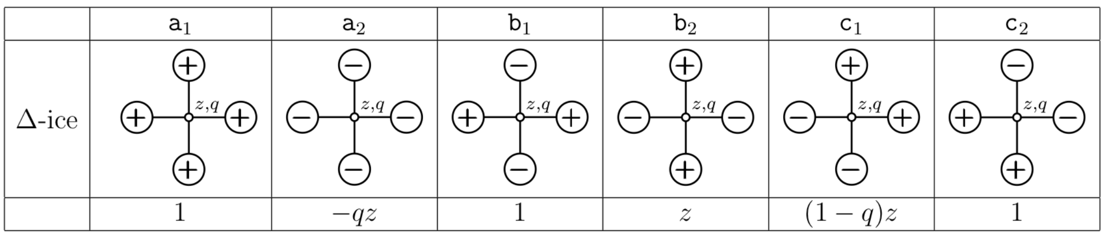
THEOREM OF BRUBAKER SCHULTZ

BOTH CURRENT OPERATORS:

J_k CHANGES ENERGY OF A PARTICLE BY $-k$

HAMILTONIAN: $H(z) = \sum_{k=0}^{\infty} \frac{1}{k}(1-q^k) z^k J_k$

AND ROW TRANSFER MATRICES FOR Δ ICE:



CAN BE THOUGHT OF AS RIGHT MOVING OPERATORS ON FERMIONIC FOCK SPACE:

WE ARE PROVING

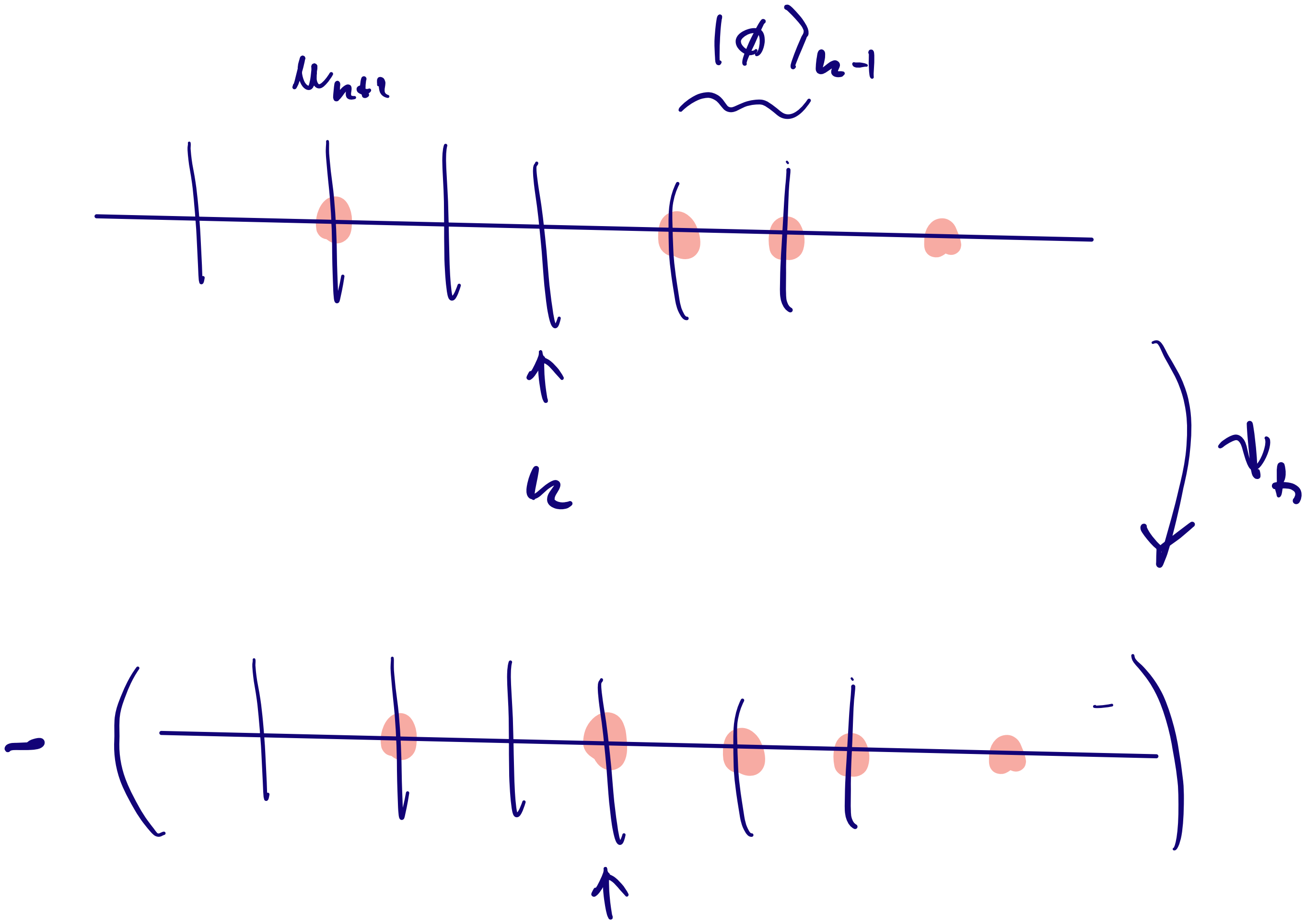
$$T(z) = e^{H(z)}.$$

↑

ROW TRANSFER

WE INTRODUCED FERMIONIC CREATION OPS,

$$\psi_n^*(\xi) = u_n \wedge \xi$$



$$\xi = u_{n+2} \wedge |\emptyset\rangle_{n-1}$$

$$\psi_n^*(\xi) = u_n \wedge u_{n+2} \wedge u_{n-1} \wedge u_{n-2} \wedge \cdots$$

$$= -u_{n+2} \wedge u_n \wedge u_{n-1} \wedge \cdots$$

$\mathcal{F}_m$ = CHARGE m FOCK SPACE

$$T(z),\ J_n : \mathcal{F}_m \to \mathcal{F}_m$$

$$\mathcal{F}_m \underset{\psi_n}{\overset{\psi_n^*}{\rightleftarrows}} \mathcal{F}_{m+1}$$

WE PROVED WEDNESDAY

$$\psi^*(x) = \sum x^k \psi_k^*$$

$$e^{H(z)}\,\psi^*(x)\,e^{-H(z)} = \frac{1-qxz}{1-xz}\,\psi^*(x).$$

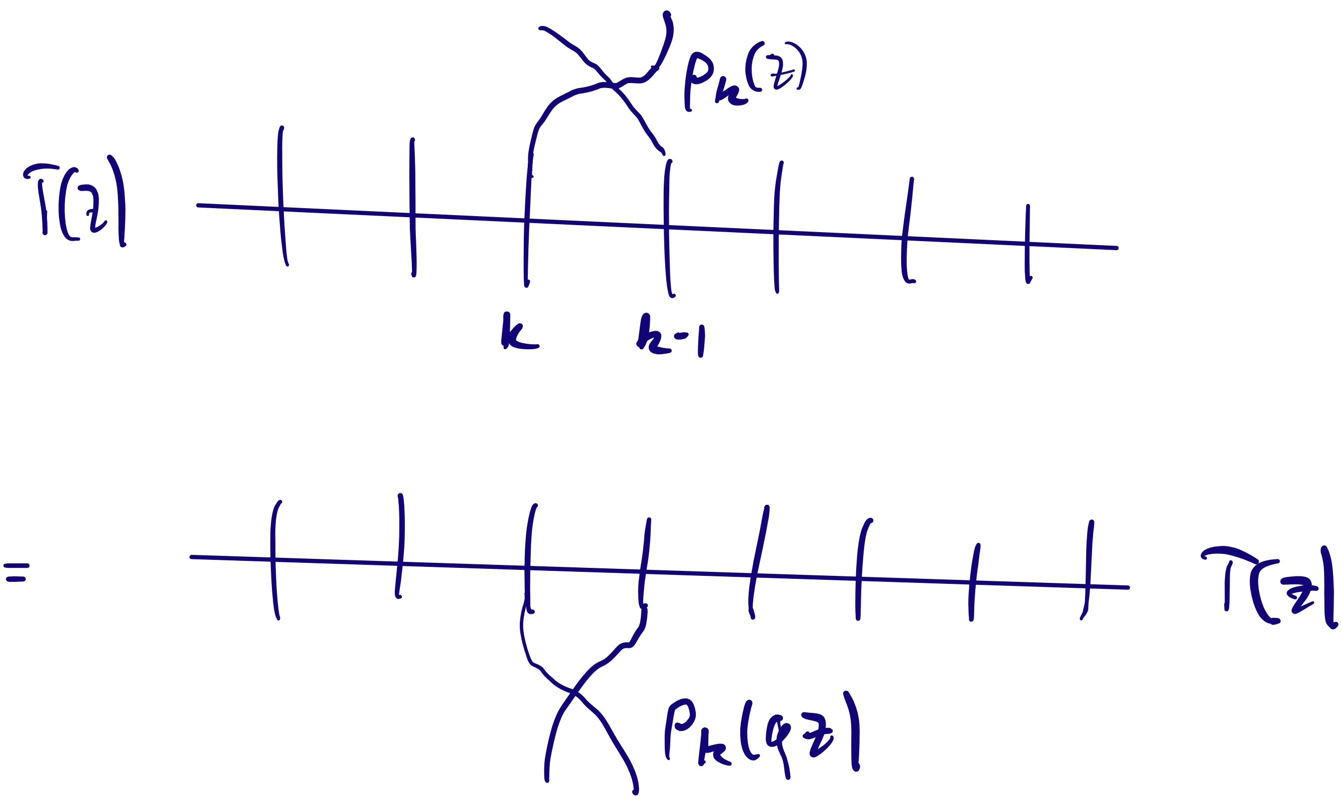
THE METHOD OF PROOF IS TO SHOW

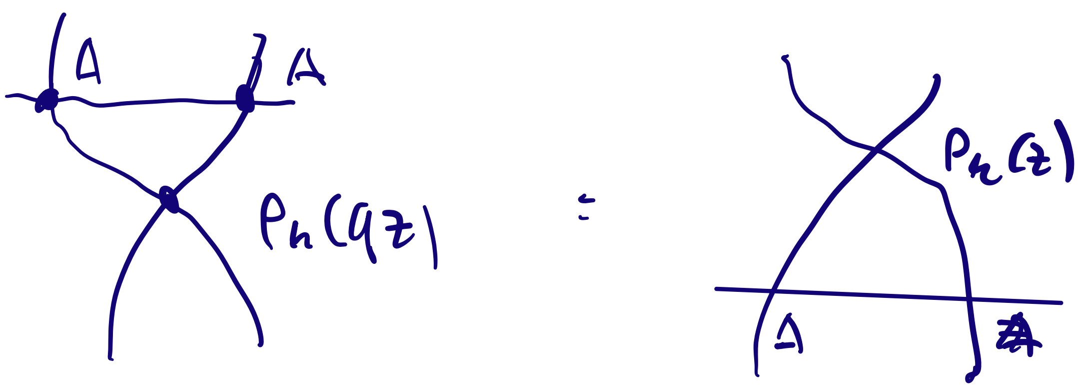
$$T(z)\,\psi^*(x)\,T(z)^{-1} = \frac{1-qxz}{1-xz}\,\psi^*(x).$$

THIS IMPLIES $T(z) = e^{H(z)}$.

WE'LL SHOW THIS IDENTITY IS A KIND OF YBE.

INTRODUCE $\quad P_k(z) = \psi_k^* - z\,\psi_{k-1}^*$





$$\eta(z) = \psi_k - z\,\psi_{k-1}$$

$$T(z)\,\psi^*(x) = \frac{1-qxz}{1-xz}\,\psi^*(x)\,T(z)$$

$$(1-xz)\,T(z)\sum x^k \psi_k^* = (1-qxz)\sum x^k \psi_k^*\,T(z)$$

COEFFICIENT OF x^k IS:

LHS $\quad T(z)\,\psi_k^* - z\,T(z)\,\psi_{k-1}^* = T(z)\,\eta_k(z)$

RHS $\quad \psi_k^*\,T(z) - qz\,\psi_k^*\,T(z) = \eta_k(qz)\,T(z).$

COMPARING, THE IDENTITY

$$T(z)\,\psi^*(x)\,T(z)^{-1} = \frac{1-qxz}{1-xz}\,\psi^*(x)$$

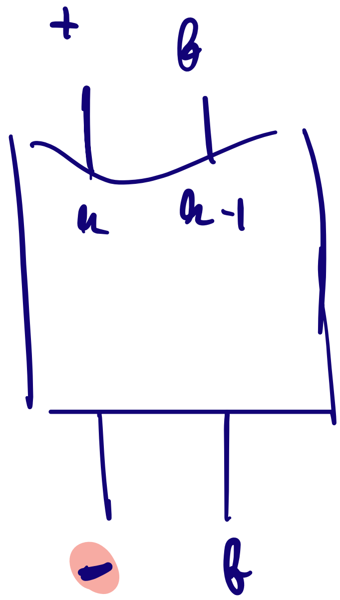
IS EQUIV. TO

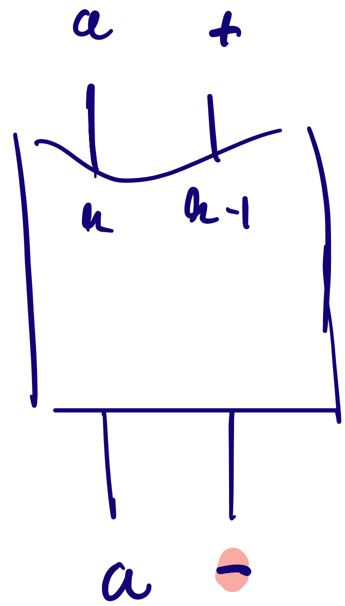
$$T(z)\,\eta_k(z) = \eta_k(qz)\,T(z)$$

$+$ = ABSENCE OF [illegible]

$-$ = OCCUPIED STATE

$\eta_n(z)$ LOOKS LIKE





$b = \pm$

1

$-z$

MORAL: IDENTITY

$$T(z)\,\psi^+(x)\,T(z)^{-1} = \left(\frac{1-qxz}{1-xz}\right)\psi^+(z)$$

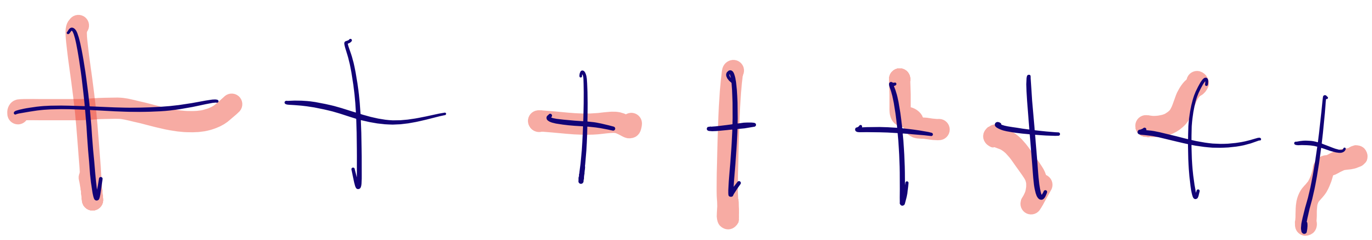
BOILS DOWN TO A YBE.

HEISENBERG SPIN CHAINS

OBTAINED SOLVABILITY OF 8-VERTEX MODEL AND HEISENBERG XYZ HAMILTONIAN USING ELLIPTIC FUNCTIONS.

EQUIVALENCE OF THESE TWO PROBLEMS SIMILAR TO THE RESULT JUST PROVED.

ROW TRANSFER MATRIX FOR 8VM:

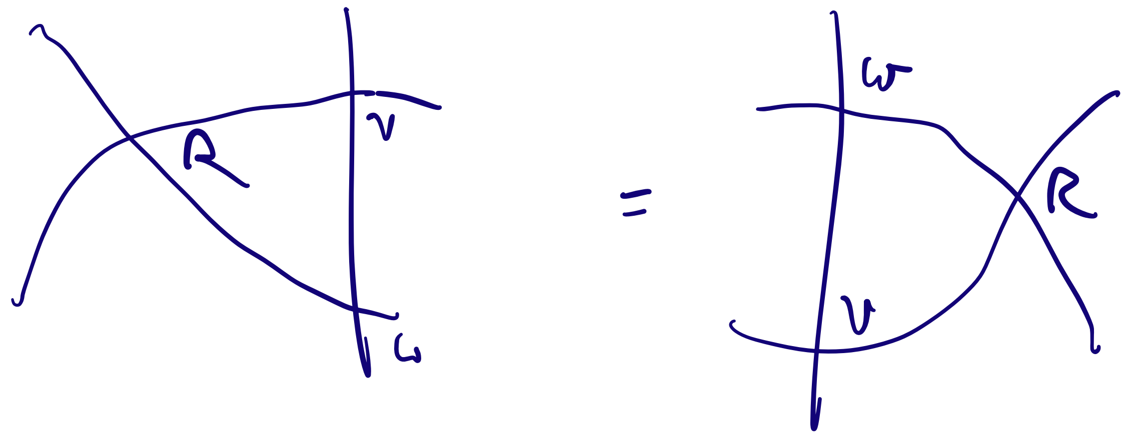


a a b b c c d d

$a = a(v) \quad b = b(v), \ldots$

COMMUTING FAMILIES OF ROW TRANSFER MATS.

GIVEN v, w WHEN IS THERE AN R SUCH THAT:



$$\Delta = \Delta(v) = \frac{a^2+b^2-c^2-d^2}{ab+cd}$$

$$\Gamma = \Gamma(v) = \frac{ab-cd}{ab+cd}.$$

THEOREM: R EXISTS IF

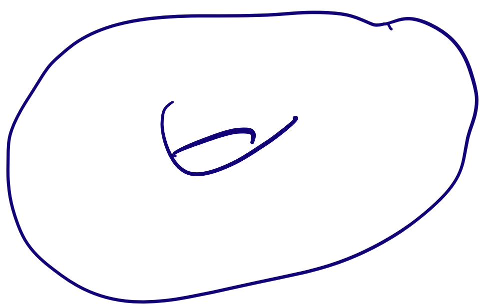
$$\Delta(w) = \Delta(v) \qquad \Gamma(w) = \Gamma(v)$$

AND IF SO $\Delta(R)$, $\Gamma(R)$ = SAME VALUES.

WITH Δ AND Γ FIXED COMPLEX NUMBERS

$$\left\{ (a, b, c, d) \;\middle|\; \frac{a^2+b^2-c^2-d^2}{ab+cd} = \Delta, \; - = \Gamma \right\}$$

FORM AN ELLIPTIC CURVE. (IT IS A GROUP.)



SO WE HAVE A PARAMETERIZED YBE WITH PARAMETER GROUP AN ELLIPTIC CURVE.

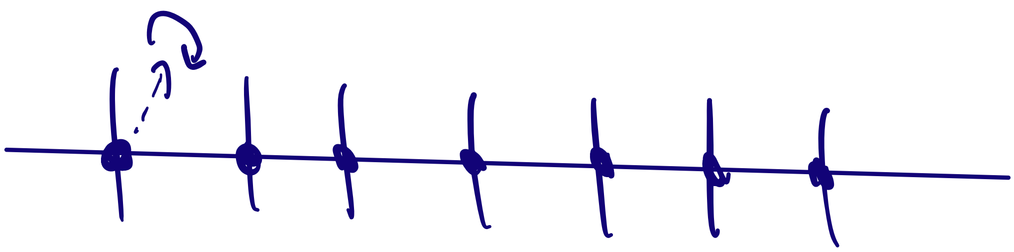
HEISENBERG SPIN CHAINS.

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^z = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$$

PAULI MATRICES HERMITIAN ON $\mathbb{C}^2$.

$$\sigma^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

MODEL OF ONE DIMENSIONAL FERROMAGNETISM.



THERE IS A TENDENCY FOR ADJACENT ATOMS TO SPIN THE SAME WAY.

EACH ATOM COMES WITH HILBERT SPACE $\mathbb{C}^2$ REPRESENTING ITS ANGULAR MOMENTUM.

$\bigotimes^N \mathbb{C}^2$ = HILBERT SPACE FOR N ATOMS.

HAMILTONIAN IS AN OPERATOR WHOSE EIGENVALUES REPRESENT ENERGY OF A CONFIGURATION.

$$H = \frac{1}{2}\sum_k J_x \sigma_k^x \otimes \sigma_{k+1}^x + J_y \sigma_k^y \otimes \sigma_{k+1}^y + J_z \sigma_k^z \otimes \sigma_{k+1}^z$$

J_x, J_y, J_z ARE CONSTANTS

BAXTER (AND OTHERS e.g. SUTHERLAND)
KNEW THESE TWO PROBLEMS WERE EQUIVALENT:
INTEGRABILITY OF 8VM AND OF HSC.

IF X = Y THIS IS CALLED XXZ
HAMILTONIAN (EQUIVALENT TO SIX VERTEX MODEL)

6 VERTEX MODEL $\quad d = 0 \quad X = Y$.

$\theta \in (0, 2\pi) \quad \chi$ FIXED.

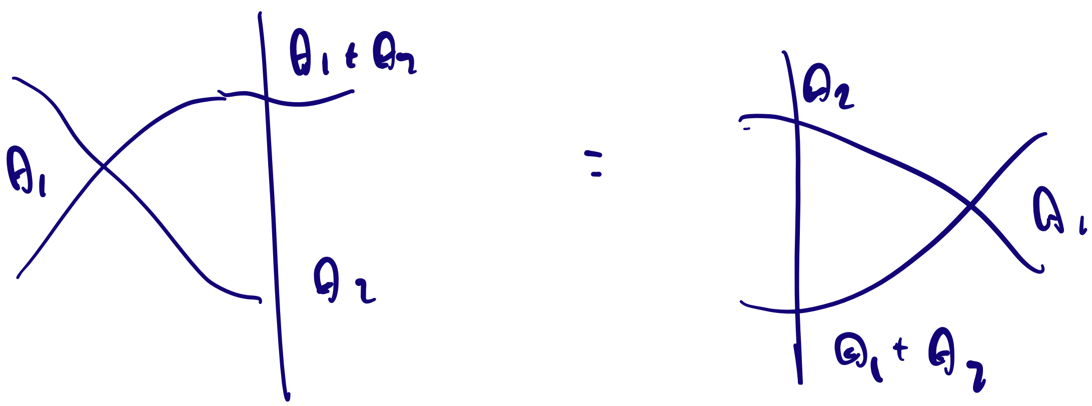
$e^{i\theta} = x$ VARIABLE. $\quad e^{i\chi} = q$ FIXED.

PARAMETRIZE A VERTEX

$$a = \frac{xq - (xq)^{-1}}{q - q^{-1}} \qquad b = \frac{x - x^{-1}}{q - q^{-1}} \qquad c = 1$$

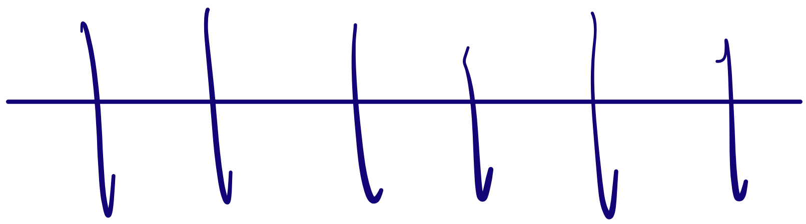
THM IF $v = v_\theta$ HAS THESE COORDS

i.e. $[[v_{\theta_1}, v_{\theta_1+\theta_2}, v_{\theta_2}]] = 0$



PARAMETRIZED YBE.

ROW TRANSFER MATRICES

 $T(\theta)$

FORM A COMMUTATIVE FAMILY.

THE RELATIONSHIP WITH SPIN CHAIN:

$$\mathcal{H}_{XXZ} + \text{const} \times I = \text{LOGARITHMIC DERIVATIVE OF } T_\theta$$

NEAR $\theta = \chi$ (IDENTITY IN PARAMETER GROUP).