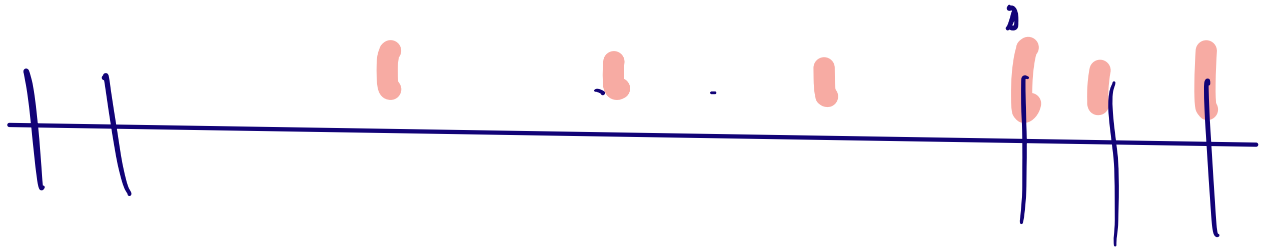
ROW TRANSFER MATRICES FOR INFINITE GRIDS:



ON A HORIZONTAL EDGE

● : OCCUPIED STATE

+ : UNOCCUPIED.

u_j : OCCUPIED STATE OF ENERGY j (j = COLUMN NUMBER)

CONFIGURATION OF SPINS ON A VERTICAL EDGE:

ALL STATES IN COLUMNS j SUFF. NEG. ARE OCCUPIED

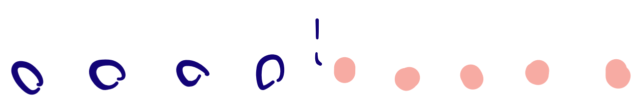
ALL STATES WITH $j \gg 0$ UNOCCUPIED

RESEMBLES DIRAC'S ELECTRON SEA = FERMIONIC FOCK SPACE

$$\mathcal{F}_m = \text{SPAN OF } u_{j_1} \wedge u_{j_2} \wedge \cdots = |j\rangle$$

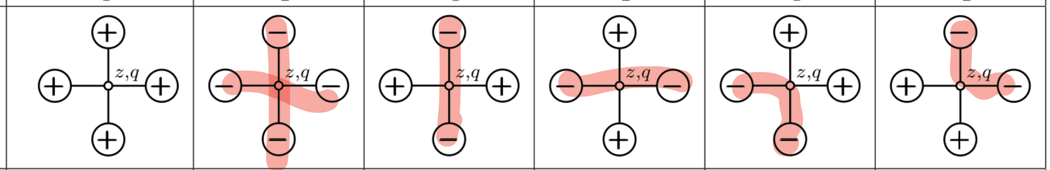
$$j_1 > j_2 > j_3 > \cdots \qquad j_k = k \text{ IF } k \ll 0$$

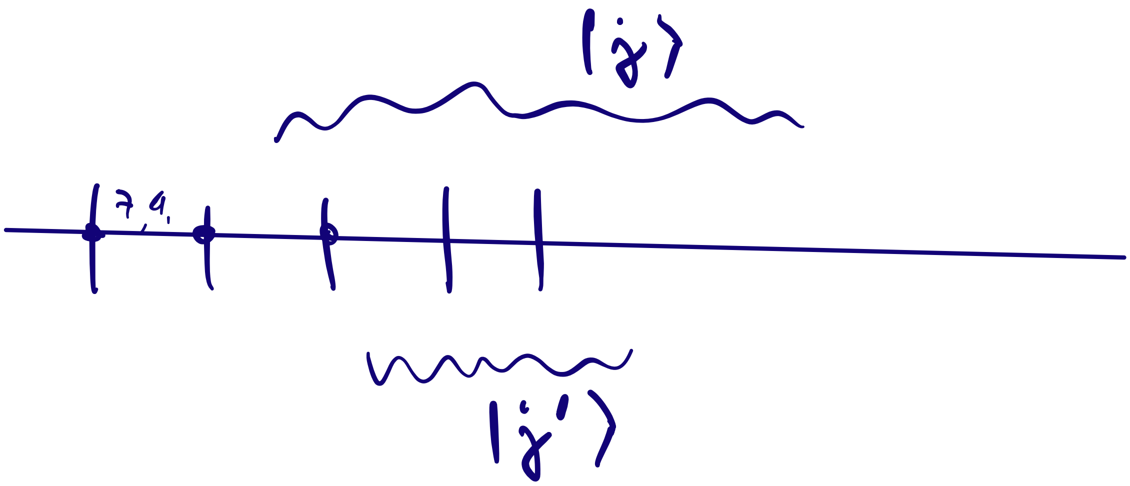
$$|0\rangle_m = u_m \wedge u_{m-1} \wedge u_{m-2} \wedge \cdots$$



GOAL IS TO INTERPRET Δ ICE

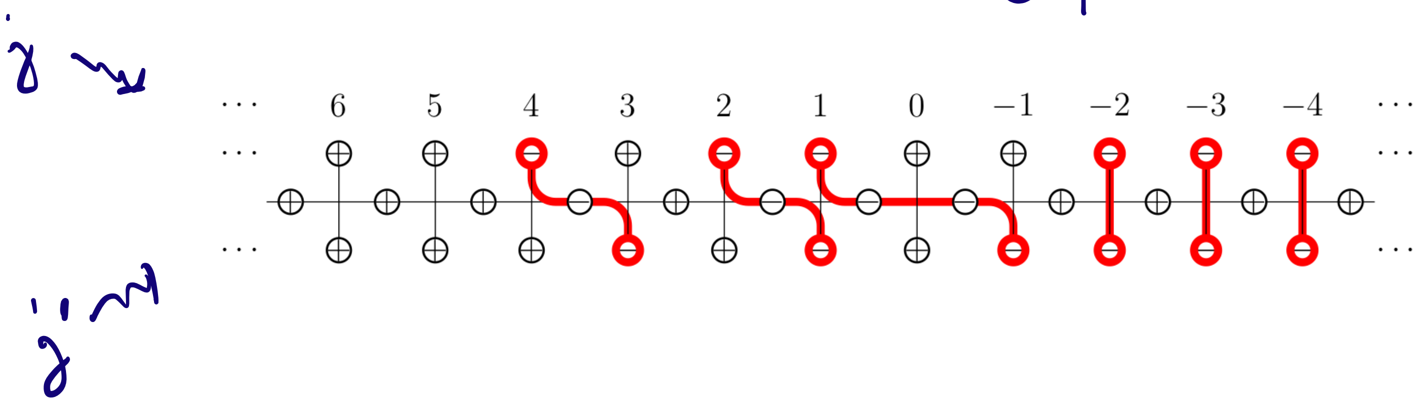
	a_1	a_2	b_1	b_2	c_1	c_2
Δ-ice						
	1	$-qz$	1	z	$(1-q)z$	1





PARTITION FUNCTION $= \prod$ BOLTZMANN WEIGHTS

ALL BUT MANY FACTORS ARE 1.

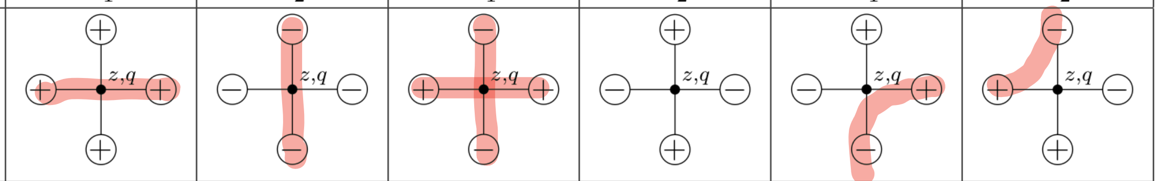


$$\langle j' | T_{z,q} | j \rangle$$ DIRAC'S NOTATION.

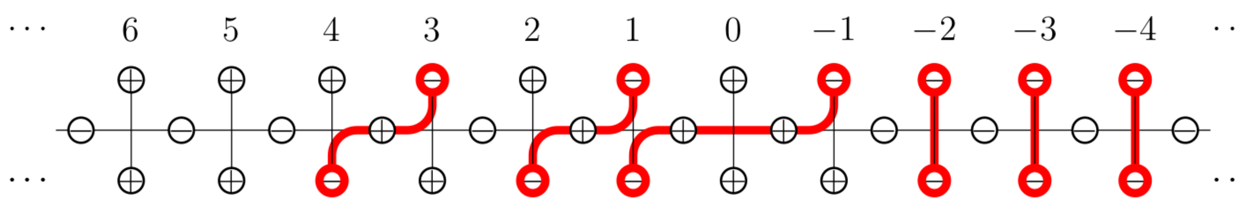
$T_{z,q}$ IS AN OPERATOR

$$T_{z,h}|j\rangle = \sum_{j'} \langle j'|T|j\rangle \, |j'\rangle .$$

THERE IS ALSO GAMMA ICE

	$\mathtt{a}_1$	$\mathtt{a}_2$	$\mathtt{b}_1$	$\mathtt{b}_2$	$\mathtt{c}_1$	$\mathtt{c}_2$
Γ-ice						
	z^{-1}	1	$-qz^{-1}$	1	$1-q$	z^{-1}

SIMILARLY A FINITE PRODUCT



IN NOTES LINES MOVE ON + SPINS FOR HORIZONTAL EDGES IN GAMMA ICE.

DELTA ICE : RIGHT MOVING STATES

GAMMA ICE : LEFT-MOVING

SIMILARLY CONSIDER Γ ROW TRANSFER MATRIX TO BE SOMETHING LIKE AN OPERATOR

$$T_\Gamma(z)\,|j\rangle = \text{INFINITE SUM}$$

PROBLEMATIC BUT STILL

$$\langle j'|T_\Gamma(z)|j\rangle$$

IS A FINITE SUM SO EVEN THOUGH $T_\Gamma(z)$ IS NOT A HILBERT SPACE OPERATOR IN USUAL WAY, NO REAL PROBLEM IF WE INTERPRET $\langle j'|T|j\rangle$ AS THE REAL OBJECTS OF INTEREST.

$$J_k u_j = u_{j-k}$$

THEOREM
$$T_A(z) = e^{H^+(z)}$$
$$T_P(z) = e^{H^-(z)}$$

$$H^+(z) = \sum_{n=1}^{\infty} \frac{1}{n}(1-q^n) z^n J_n$$

$$H^-(z) = \sum \frac{1}{k}(1-q^k) z^{-n} J_{-n}$$

OPERATORS SUCH AS

$$e^{H^-(z)} \quad e^{H^+(z)}$$ DON'T COMMUTE

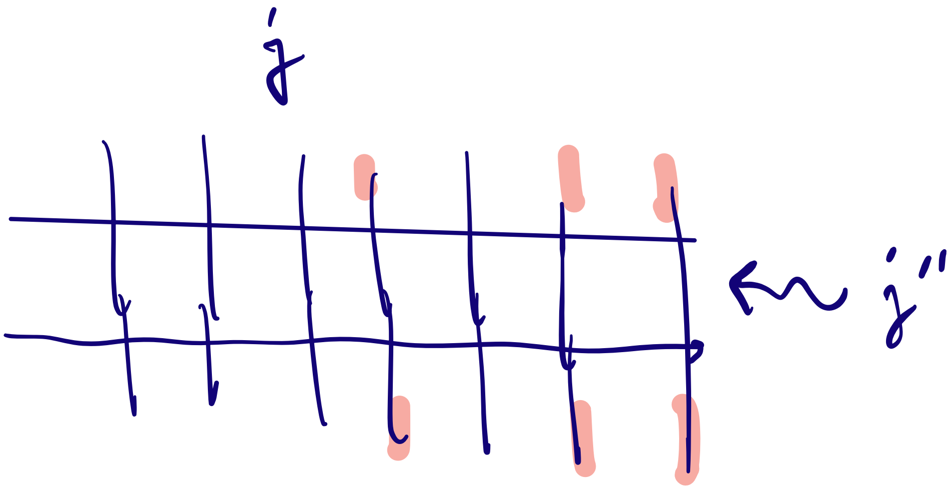
ARE CALLED VERTEX OPERATORS... THEY OCCUR IN CONFORMAL FIELD THY, SOLITON THY.

$$T_A(z)\, T_A(w) = T_A(w)\, T_A(z).$$

CAN BE PROVED USING YBE.

THIS MEANS

$$\langle j' | T_A(z)\, T_A(w) | j \rangle = \cdots$$



j'

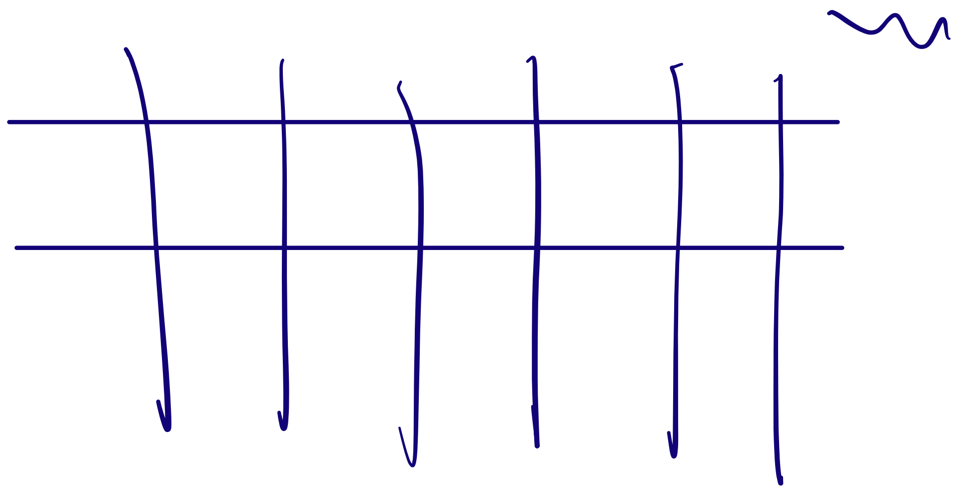
$$\sum_{j''} \langle j' | T(z) | j'' \rangle \langle j'' | T_A(w) | j \rangle$$

TRUNCATE GRID SO ALL STATES OF

j OR j' (THEREFORE OF j'')

ARE OCCUPIED RIGHT OF WHERE WE CUT

UNOCCUPIED TO LEFT.



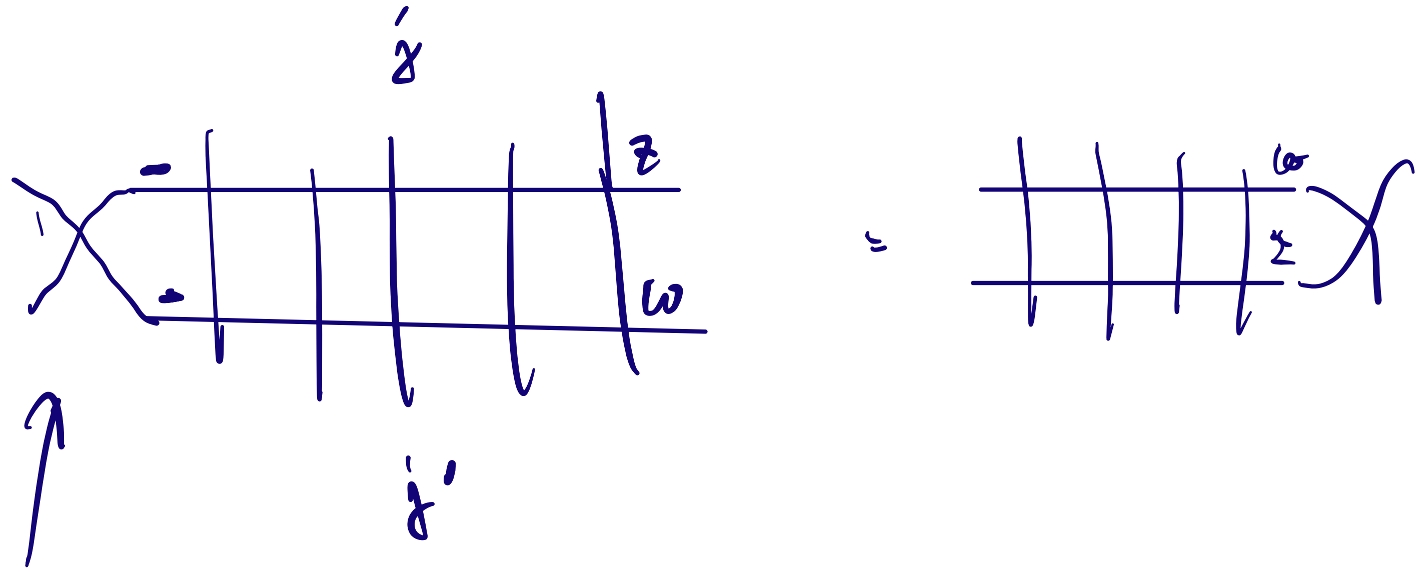
ALL VERTICES LEFT OR RIGHT OF THIS

PART HAVE a_1 OR b_1 CONFIGURATIONS

THIS DOESN'T CHANGE VALUE.

WE CAN ATTACH R-MATRIX, RUN TRAIN

ARGUMENT;



MULTIPLIES
P.F. BY
A CONSTANT.

SO $\langle j' | T(z) T(w) | j \rangle = \langle \; T(w) | T(z) \; \rangle$

FERMIONIC CREATION OPERATION OPS:

ψ_k^* CREATES A PARTICLE AT LEVEL k

$$\psi_k^*(\xi) = u_k \wedge \xi$$

$$\xi = u_{j_m} \wedge u_{j_{m-1}} \wedge \dots$$

$$\psi_k^*(\xi) = 0 \quad \text{IF} \quad k \in \{ j_m, j_{m-1}, \dots \}$$

$$\pm | j' \rangle \quad \text{WHERE}$$

$$\{ j'_{m+1}, j'_m, j'_{m-1}, \dots \} = \{ j_m, j_{m-1}, \dots \} \cup \{k\}$$

THEIR ADJOINTS ψ_k ARE ANNIHILATION OPERATORS.

$$\mathcal{F}_m \underset{\psi_k}{\overset{\psi_k^*}{\rightleftarrows}} \mathcal{F}_{m+1}$$

THEY SATISFY $\psi_k^* \psi_\ell + \psi_\ell \psi_k^* = \delta_{k,\ell}$ CLIFFORD ALGEBRA.

$$[J_k, \psi_m^*] = \psi_{m-k}^*$$

$$H(z) = \sum_{k=1}^{\infty} \frac{1}{k}(1-q^k) z^k$$

$$[H(z), \psi_m^*] = \sum \frac{1}{k}(1-q^k) z^k \psi_{m-k}^* .$$

WE CAN REPACKAGE THIS BY INTRODUCING FERMIONIC FIELD

$$\psi^*(x) = \sum_{-\infty}^{\infty} x^k \psi_k^*$$

(FORMAL GADGET.)

$$[H(z), \psi^*(x)] = \log\left(\frac{1-qxz}{1-xz}\right) \psi^*(x).$$

$$[J_k, \psi^*(x)] = \sum x^m [J_n, \psi_m^*]$$

$$= \sum x^m \psi^*_{m-k}$$

$$= x^k \psi^*(x)$$

$$\left[\sum_{k=1}^{\infty} \frac{1}{k}(1-q^k) z^k J_k, \psi^*(x)\right] =$$

$$\sum \frac{1}{k} x^k z^k \psi^*(x) - \sum \frac{1}{k} x^k z^k q^k \psi^*(x)$$

$$= \Big(-\log(1-xz) + \log(1-qxz) \Big) \psi^*(x).$$

$$\sum \frac{1}{k} t^k = -\log(1-t)$$

$$[H, \psi^*(x)] = C\,\psi^*(x)$$

$$C = \log\left(\frac{1-qxz}{1-xz}\right)$$

EXPONENTIATE (BAKER-C.-H. FORMULA)

$$e^H \cdot \psi^*(x)\, e^{-H} = e^C \cdot \psi^*(x).$$

STRATEGY: SHOW USING A KIND OF COLUMN YBE THAT $T_A(z)$ SATISFIES ALS

$$T(z)\,\psi^*(x)\,T(z)^{-1} = \left(\frac{1-qxz}{1-xz}\right)\psi^*(x).$$