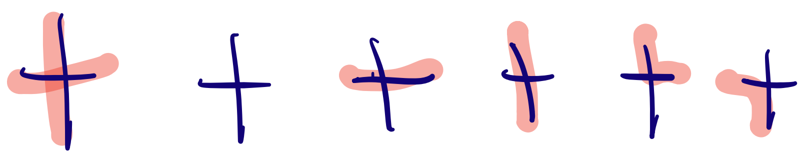
INTERPRETATION OF ROW TRANSFER MATRICES FOR $\Gamma$ AND $\Delta$ "FREE-FERMIONIC" AS VERTEX OPERATORS.

THERE IS A DICHOTOMY IN 6 VERTEX MODEL



$a_1$ $a_2$ $b_1$ $b_2$ $c_1$ $c_2$

+ = NO COLOR

= RED

- MODELS RELATED TO $U_q(\mathfrak{gl}(1|1))$
  "FREE-FERMIONIC"

$$a_1 a_2 + b_1 b_2 = c_1 c_2$$

TOKUYAMA MODELS ARE FREE FERMIONIC.

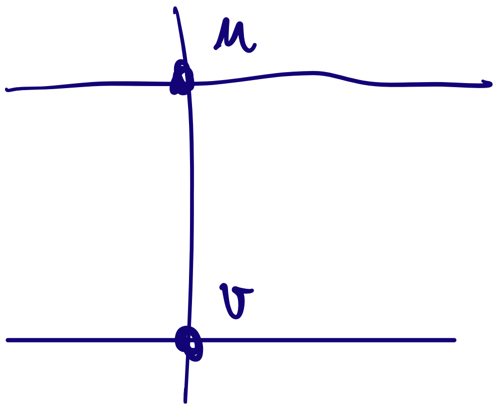
- MODELS RELATED TO $U_q(\mathfrak{gl}(2))$
  CAN ARRANGE $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$ "FIELD FREE"

$$\frac{a^2 + b^2 - c^2}{2ab} = \underset{\text{BAXTER}}{\Delta} = \frac{1}{2}(q + q^{-1}).$$

$a = a(v) \quad b = b(v) \quad c = c(v)$

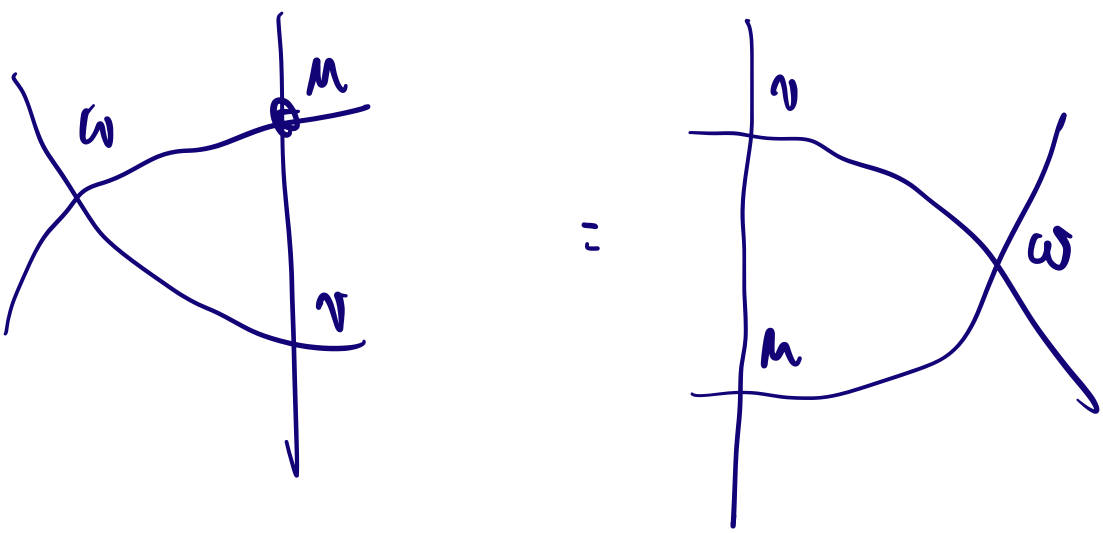
$\Delta = \Delta(v)$

BAXTER PROVED IF $\Delta(u) = \Delta(v)$




THERE IS A $w$ WITH $\Delta(w) = \Delta(u) = \Delta(v)$

S.T. YBE IS SATISFIED




FURTHERMORE HE EXTENDED THIS THEORY TO EIGHT VERTEX MODEL.

IN THE GENERALITY OF 8 VM HE PROVED A RELATIONSHIP BETWEEN HAMILTONIANS OF THE HEISENBERG SPIN CHAINS AND ROW TRANSFER MATRICES FOR 8 VM.

PROVE HEISENBERG SPIN CHAINS ARE INTEGRABLE.

LATER PAUL ZINN-JUSTIN ($g=0$) AND BRUBAKER-SCHULTZ (GENERAL $g$) PROVED A SIMILAR RELATIONSHIP BETWEEN ROW TRANSFER MATRICES FOR TOKUYAMA MODEL AND CERTAIN HAMILTONIANS.

THIS I'LL TALK ABOUT B.-S. THM.

FERMIONIC FOCK SPACE WAS INTRODUCED BY DIRAC IN THEOREM OF THE ELECTRON.

THE DIRAC EQUATION IS SOME DIFF'L EQ'N

WITH SOLUTIONS OF VARIOUS ENERGY LEVELS

$k \in \mathbb{Z}$. $\qquad u_k$ = SOLUTIONS.

IT SEEMED PROBLEMATIC THAT SOLUTIONS CAN HAVE ARBITRARILY NEG. ENERGIES.

SOLUTIONS ARE FERMIONS THEIR ALGEBRA BEHAVES LIKE AN EXTERIOR ALGEBRA

$$u_h \wedge u_l = -u_l \wedge u_k.$$

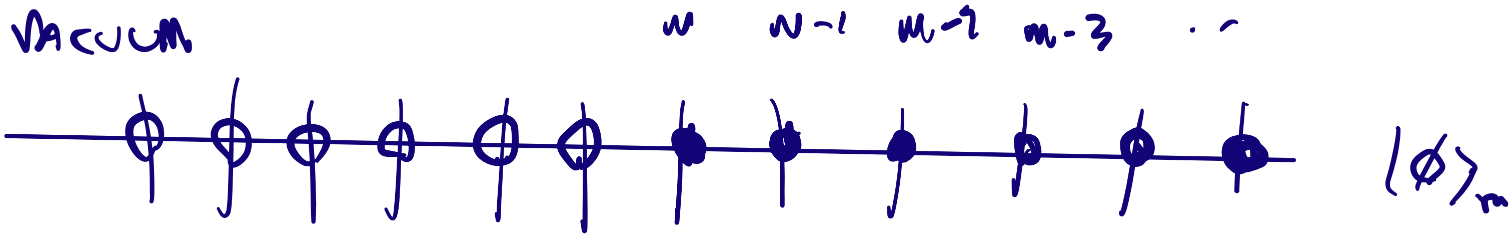
PAULI EXCLUSION PRINCIPLE: TWO FERMIONS CANNOT OCCUPY SAME STATE.

SO THERE IS A HILBERT SPACE OF SOL'NS OF FORM

$$|j\rangle_m = u_{j_m} \wedge u_{j_{m-1}} \wedge u_{m-2} \wedge \cdots$$

$$j_m > j_{m-1} > j_{m-2} > \cdots$$

$$j_k = k \text{ IF } k << 0.$$

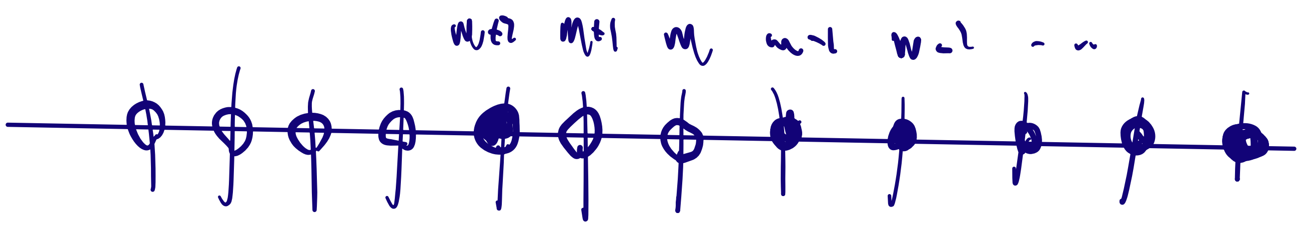



MAYA DIAGRAM. ● = OCCUPIED STATE

a $j_n = a$ SAME $a$

○ = UNOCCUPIED STATE

ALL SUFFICIENTLY NEGATIVE STATES ARE OCCUPIED.




$$|j\rangle_m = u_{m+2} \wedge u_{m-1} \wedge u_{m-2} \wedge \cdots$$

"SEMI-INFINITE MONOMIAL"

SPAN OF ALL $|j\rangle_m$ IS $\mathcal{F}_m$

FERMIONIC FOCK SPACE OF CHARGE $m$.

$J_k$ IS AN OPERATOR ON $\mathfrak{g}_m$

MOVES A PARTICLE BY $-k$.

$$J_k u_j = u_{j-k}$$

APPLIED TO WEDGES IT IS DISTRIBUTED BY LEIBNITZ RULE

$$J_k ( u_{j_m} \wedge u_{j_{m-1}} \wedge \cdots ) =$$

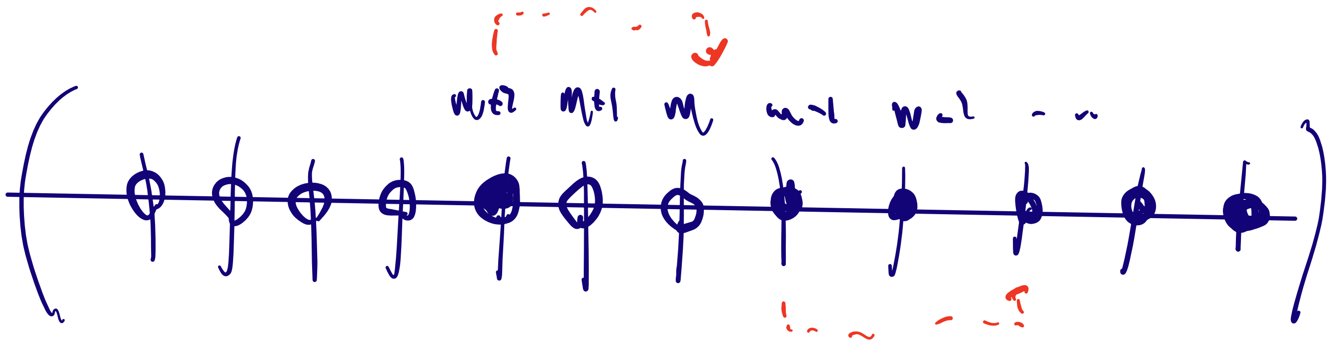
$$\sum_{l \leq m} ( u_{j_m} \wedge u_{j_{m-1}} \wedge \cdots \wedge u_{j_l - k} \wedge \cdots )$$

THIS IS REALLY A FINITE SUM.

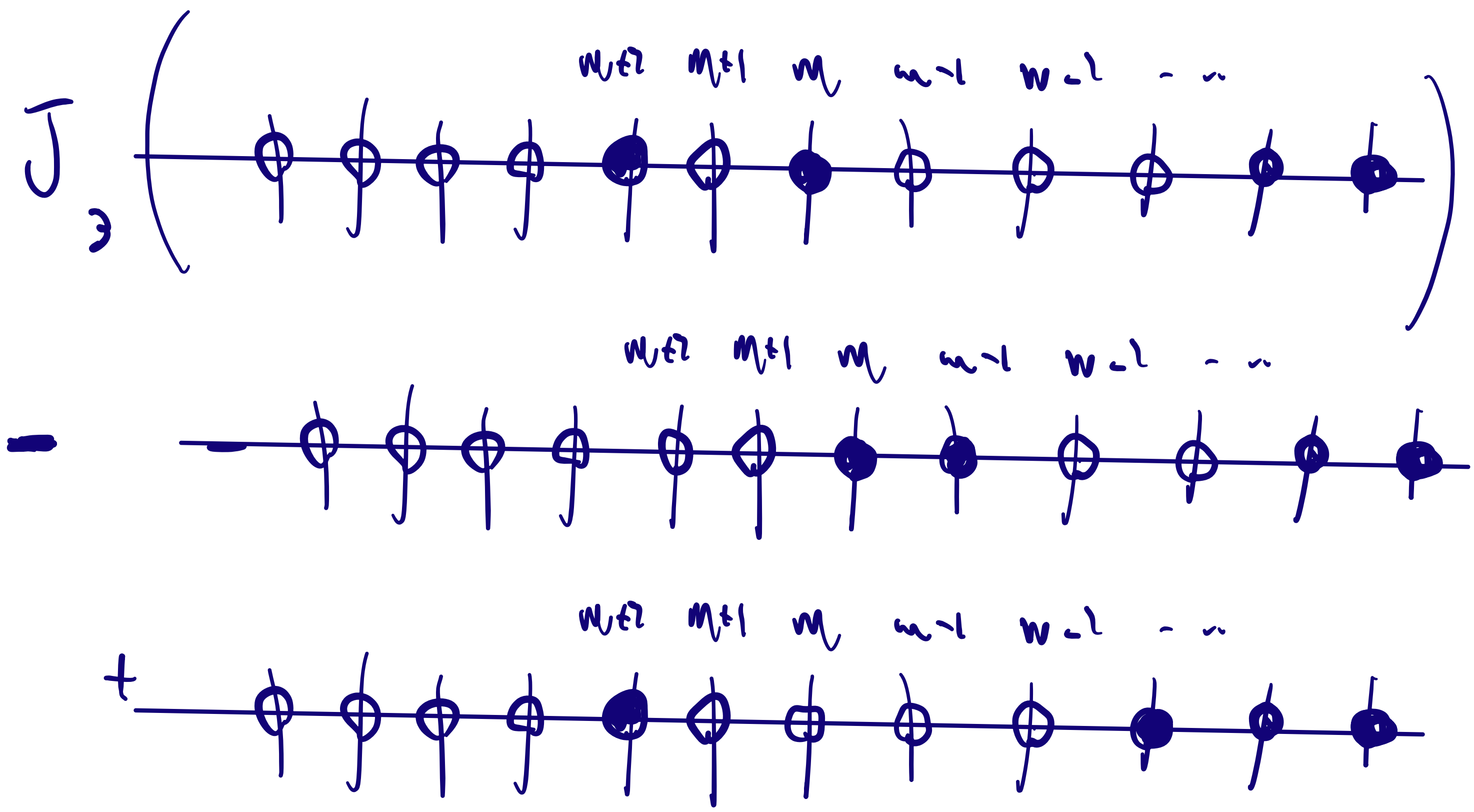
IT IS UNDERSTOOD $u_a \wedge u_b = -u_b \wedge u_a$

$u_a \wedge u_a = 0$

ONLY FINITELY MANY TERMS ARE NONZERO.

$J_2$ 


$= |\emptyset\rangle_m$

$J_3$ 


THE FERMIONIC FOCK SPACE LEAD TO INTERESTING CONNECTIONS WITH (FOR EXAMPLE\ REP'N THEORY OF SYMMETRIC GROUP, HEISENBERG LIE ALG, ...

**THM**: $[J_k, J_l] = k \cdot I$ IF $l = -k$

$0$ OTHERWISE

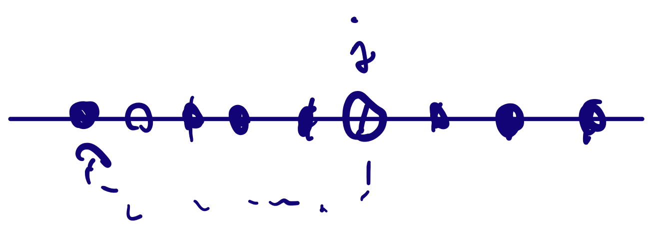
$\mathcal{H} = \bigoplus \mathbb{C} J_k \oplus \mathbb{C} \cdot I$ IS A HEISENBERG LIE ALGEBRA.

$Z(\mathcal{H}) = \mathbb{C} \cdot I$ ⟵ CENTRAL.

$$(J_k J_{-k} - J_{-k} J_k)|\emptyset\rangle = k|\emptyset\rangle$$

IT IS EQUIV. TO PROVE FOR $k$ OR $-k$
SO WLOG $k > 0$.

$J_{-k}|\emptyset\rangle$ = SUM OF $k$ STATES




$m = 0$

$j \overset{?}{=} -k$

APPLYING $J_k$ HAS TO PUT THIS PARTICLE BACK.

PROVING $[J_n, J_{-n}]\,|\emptyset\rangle$ EASY, FOR MORE GENERAL $|j\rangle$, AN EXERCISE.

A BASIS FOR $\mathcal{F}_m$ CONSISTS OF

$$|\lambda\rangle = u_{\lambda_1+m} \wedge u_{\lambda_2+m-1} \wedge \cdots$$

$\lambda = (\lambda_1, \lambda_2, \dots)$ A PARTITION.

V. KAC: BOSON - FERMION CORRESPONDENCE RELATES THIS VECTOR TO SCHUR FUNCTIONS.

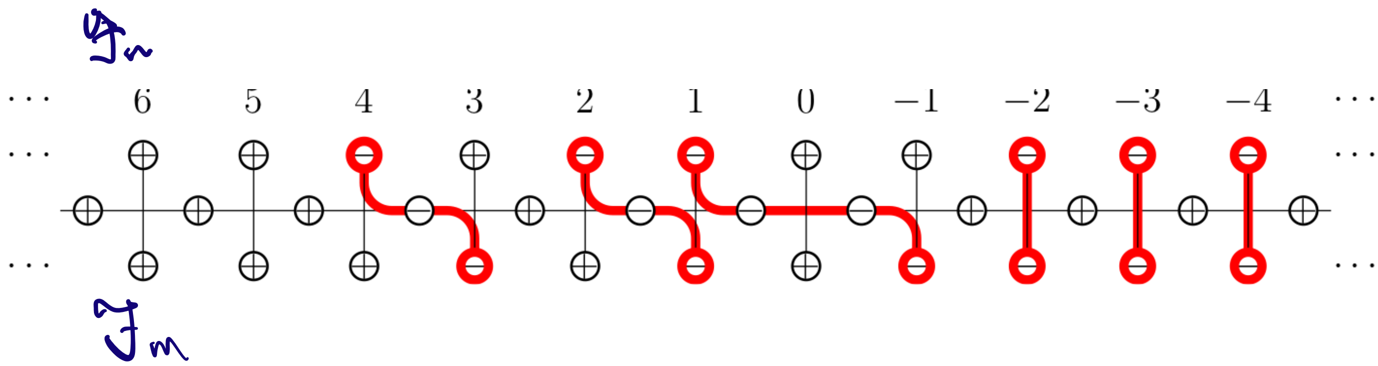
DELTA ICE: FROM TOKUYAMA MODELS

| | $a_1$ | $a_2$ | $b_1$ | $b_2$ | $c_1$ | $c_2$ |
|---|---|---|---|---|---|---|
| $\Delta$-ice | $\begin{smallmatrix} & + & \\ + & z,q & + \\ & + & \end{smallmatrix}$ | $\begin{smallmatrix} & - & \\ - & z,q & - \\ & - & \end{smallmatrix}$ | $\begin{smallmatrix} & - & \\ + & z,q & + \\ & - & \end{smallmatrix}$ | $\begin{smallmatrix} & + & \\ - & z,q & - \\ & + & \end{smallmatrix}$ | $\begin{smallmatrix} & + & \\ - & z,q & + \\ & - & \end{smallmatrix}$ | $\begin{smallmatrix} & - & \\ + & z,q & - \\ & + & \end{smallmatrix}$ |
| | $1$ | $-qz$ | $1$ | $z$ | $(1-q)z$ | $1$ |

$q$, $z$ SOME COMPLEX NUMBER

WE CONSIDER THIS AN OPERATOR ON $\psi^*_m$

INTRODUCE SPINS ON HORIZONTAL EDGES SO THAT ALL BUT FINITELY MANY HORIZONTAL SPINS ARE $\oplus$ SO ALL BUT FINITELY MANY STATES ARE $a_1$ OR $b_1$




$\ominus$ = OCCUPIED STATES

$\oplus$ = UNOCCUPIED.

THE ROW TRANSFER MATRIX IS A FINITE PRODUCT.

THEOREM:

$$H(z) = \sum_{n=1}^{\infty} \frac{1}{n}(1-q^{n})\, z^{n} J_{n}$$

THEN

$$T(z) = e^{H(z)}.$$

ROW TRANSFER MATRIX