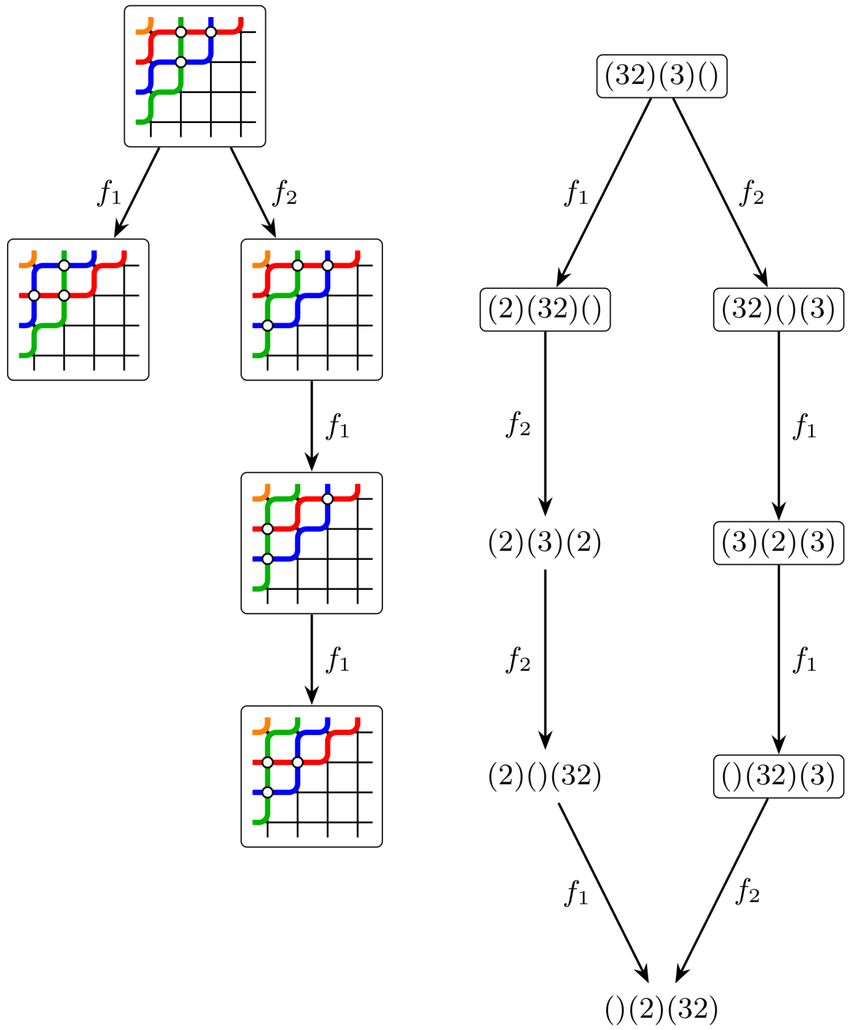
GOAL IS TO SEE THE PIPEDREAMS LIKE STATES OF TOKUYAMA $q=0$ CAN BE ORGANIZED IN DEMAZURE CRYSTALS.

$w = \Delta_3 \Delta_2 \Delta_3$

$= 1432$



ONE INGREDIENT OF THIS STORY IS EDELMAN-GREENE INSERTION. (STARTED MONDAY.)

THIS CAN BE USED TO PROVE

THM (STANLEY): THE NUMBER OF REDUCED WORDS FOR $w_0 \in S_n$ IS $\binom{n}{2}! \big/ 1^{n-1} \cdot 3^{n-2} \cdots (2n-3)^1$.

THIS IS THE NUMBER OF STANDARD TABLEAUX OF SHAPE $\rho = (n-1, n-2, \dots, 0)$

A STANDARD TABLEAU OF SIZE k HAS STRICTLY INCREASING ROWS AND COLUMNS AND CONTAINS $1, 2, \dots, k$ WITH NO REPETITIONS.

$\rho = (2,1,0)$
$n = 3$ STANDARD TABLEAUX

1	2	1	3
3		2	

AND 2 IS ALSO THE NUMBER OF REDUCED WORDS FOR $GL(3)$

RSK TAKES A WORD AND PRODUCES A TABLEAU BY SUCCESSIVELY INSERTING ENTRIES.

IT ACTUALLY PRODUCES 2 TABLEAUX:

SECOND TABLEAU IS THE RECORDING TABLEAU.

THERE ARE n^k WORDS

$i_1, \dots, i_k \qquad i_1, \dots, i_k \in \{1, 2, \dots, n\}$

THESE CAN BE PUT IN BIJECTION WITH PAIRS OF TABLEAUX

T, T'

OF SAME SHAPE $\lambda \vdash k$ WITH T' STANDARD

T SEMISTANDARD.

$1 \ 2 \ 2 \ 1 \ 3 \ 2$

$\emptyset \leftarrow \boxed{1}$

$\boxed{1} \leftarrow 2$

$\boxed{1\ 2} \leftarrow 2$

$\boxed{1\ 2\ 2} \leftarrow 1$

$\begin{matrix} 1 & 1 & 2 \\ 2 & & \end{matrix} \leftarrow 3$

$\begin{matrix} 1 & 1 & 2 & 3 \\ 2 & & & \end{matrix} \leftarrow$

$\begin{matrix} 1 & 1 & 2 & 2 \\ 2 & 3 & & \end{matrix}$

$$\begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 5 \\ \hline 4 & 6 & \multicolumn{2}{c}{} \\ \cline{1-2} \end{array}$$

WE OBTAIN 2 TABLEAUX

$\begin{matrix} 1 & 1 & 2 & 2 \\ 2 & 3 & & \end{matrix}$ SEMISTANDARD

$\begin{matrix} 1 & 2 & 3 & 5 \\ 4 & 6 & & \end{matrix}$ STANDARD.

THEOREM (SCHENSTED, 1963):

THIS IS A BIJECTION BETWEEN WORDS $i_1, \ldots, i_k$ IN $\{1, 2, \ldots, n\}$ AND PAIRS OF TABLEAUX T, T' OF SAME SHAPE

T SSYT IN $\{1, 2, \ldots, n\}$

T' STANDARD IN $\{1, 2, \ldots, k\}$.

A VARIANT EDELMAN GREENE USEFUL WHEN THE WORD $(i_1, \ldots, i_k)$ IS A REDUCED WORD FOR $w \in S_n$.

$i_1, \ldots, i_A \in \{1, 2, \ldots, n-1\}$

If $n = 4$ there are 16 reduced words for w_0

1 2 1 3 2 1

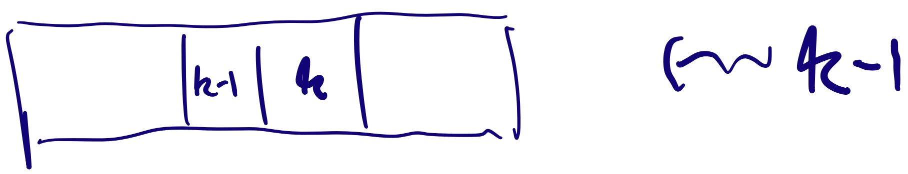
3 2 3 1 2 3

3 1 2 3 1 2

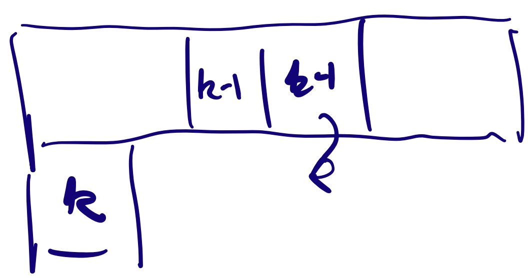
$\vdots$

Edelman Greene insertion resembles RSK but there is one exception.

Row

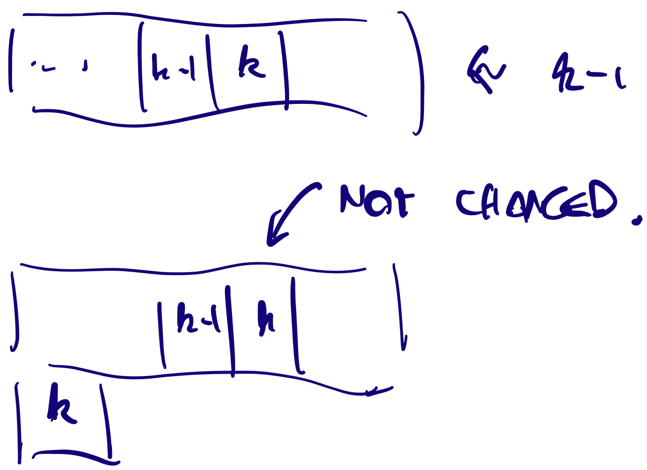


RSK would



Edelman-Greene does not change the k to $k-1$.

E.G.



$3\ 2\ 3\ 1\ 2\ 3$ $\qquad w_0 = 4321 \quad S_4.$

$\emptyset \leftarrow 3$

$3 \leftarrow 2$

$\begin{array}{l} 2 \\ 3 \end{array} \leftarrow 3$

$\begin{array}{ll} 2 & 3 \\ 3 \end{array} \leftarrow 1$

$\begin{array}{ll} 1 & 3 \\ 2 \\ 3 \end{array} \leftarrow 2$

$\begin{array}{ll} 1 & 2 \\ 2 & 3 \\ 3 \end{array} \leftarrow 3 \qquad \begin{array}{lll} 1 & 2 & 3 \\ 2 & 3 \\ 3 \end{array}$

$\begin{array}{lll} 1 & 3 & 6 \\ 2 & 5 \\ 4 \end{array}$

RECORDING TABLEAU

WHICH IS THE IMPORTANT TABLEAU HERE.

EDELMAN GREENE CHANGES THE READING WORD BUT TO ANOTHER REDUCED WORD FOR SAME PERMUTATION.

3 1 2 3 1 2

$\emptyset \leftarrow 1\,\boxed{3}$

$3 \leftarrow 1$

$\begin{matrix} 1 \\ 3 \end{matrix} \leftarrow 2$

$\begin{matrix} 1 & 2 \\ 3 & \end{matrix} \leftarrow 3$

THIS IS THE CASE EG DIFFERS FROM RSK

$\begin{matrix} 1 & 2 & 3 \\ 3 & & \end{matrix} \leftarrow 1$

$\begin{matrix} 1 & 2 & 3 \\ 2 & & \\ 3 & & \end{matrix} \leftarrow 2$

$\begin{matrix} 1 & 2 & 3 \\ 2 & 3 & \\ 3 & & \end{matrix}$

$\begin{matrix} 1 & 3 & 4 \\ 2 & 6 & \\ 5 & & \end{matrix}$

SUMMARIZE:

3 2 3 1 2 3

$$\begin{array}{ccc} 1 & 2 & 3 \\ 2 & 3 & \\ 3 & & \end{array} \qquad \begin{array}{ccc} 1 & 3 & 6 \\ 2 & 5 & \\ 4 & & \end{array}$$

T $\qquad$ T' = RECORDING TABLEAU

312312

$$\begin{array}{ccc} 1 & 2 & 3 \\ 2 & 3 & \\ 3 & & \end{array} \qquad \begin{array}{ccc} 1 & 3 & 4 \\ 2 & 6 & \\ 5 & & \end{array}$$

APPLYING EDELMAN GREENE TO ANY REDUCED WORD TO w_0 IN S_n ALWAYS PRODUCES

$$\begin{array}{ccccc} 1 & 2 & 3 & \cdots & n-1 \\ 2 & 3 & \cdots & n-1 & \\ \vdots & & & & \\ n-1 & & & & \end{array}$$

BECAUSE THIS IS THE ONLY TABLEAU WITH A REDUCED READING WORD FOR w_0 !

$$\mathfrak{J}_{w_0} = \Delta_{w_0}$$

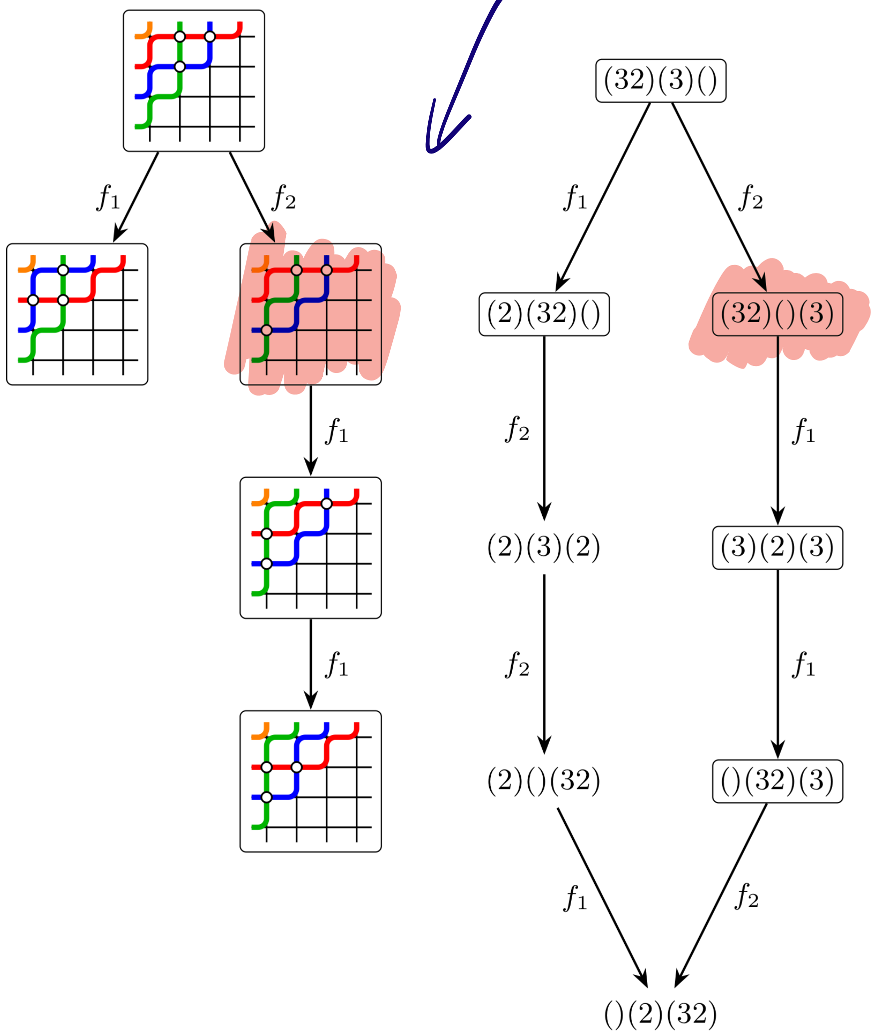
THE REDUCED WORDS FOR w_0
EQUALS THE NUMBER OF POSSIBLE
RECORDING TABLEAUX.

= # STANDARD TABLEAUX OF SHAPE

$\rho = (n-1, n-2, \dots, 0)$.

THIS IS EDELMAN GREENE'S PROOF OF
STANLEY'S THEOREM.

ANALYZED BELOW.

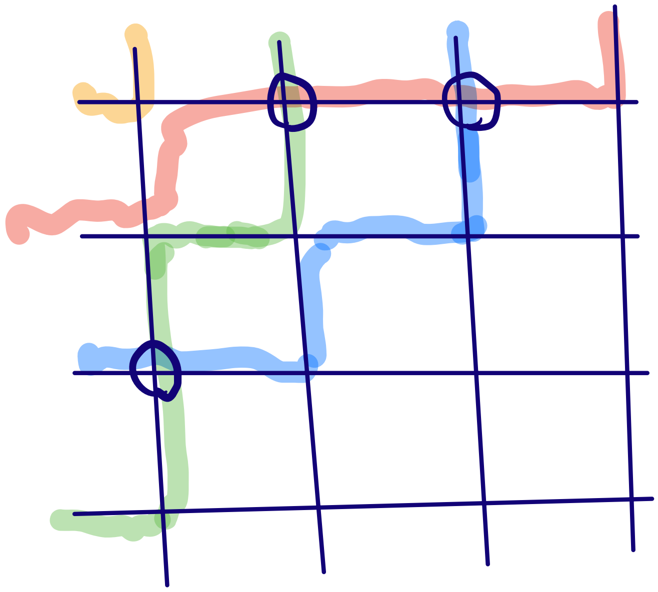


THIS SHOWS THE FIVE POSSIBLE PIPEDREAMS FOR $w = 1432 \in S_4$ $\ell(w) = 3$

SO EACH PIPEDREAM HAS 3 CROSSINGS

AND PIPEDREAM CONTRIBUTES A TERM TO THE SCHUBERT POLYNOMIAL

$\mathfrak{S}_w$

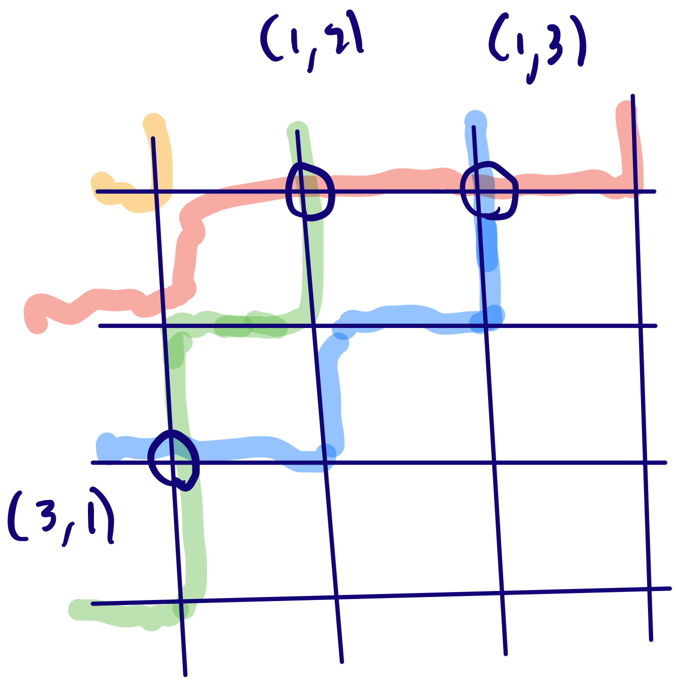


$x_1^2 x_3$

FROM THIS PIPEDREAM WE CAN READ OFF A REDUCED FOR w

IF THERE IS A CROSSING AT (i, j) LOCATION GET A REFLECTION

Δ_{i+j-1} VISIT CROSSINGS ROW BY ROW FROM RIGHT TO LEFT.



$\Delta_3 \Delta_2 \Delta_1$.

$$w_i \left\{ \begin{array}{ll} (1,3) & \Delta_3 \\ (1,2) & \Delta_2 \\ (3,1) & \Delta_3 \end{array} \right.$$

$\omega^1 = \Delta_3 \Delta_2$

$\omega^2 = 1$

$\omega_3 = \Delta_3$

IF $\omega^1, \omega^2, \ldots$

ARE THE DECREASING PERMUTATIONS

ω^n = PRODUCT OF THE REFLECTIONS COMING FROM CROSSINGS IN n-TH ROW.

$w = w^1 w^2 w^3 \quad : \quad (32)(\ )(3)$

32 MEANS $\Delta_3 \Delta_2$

EDELMAN - GREEN ACCORDING TABLEAU WILL HELP US UNDERSTAND CRYSTAL STRUCTURE

MORSE - SCHILLING

ASAF - SCHILLING

GOLD - MILICEVIC - SUN