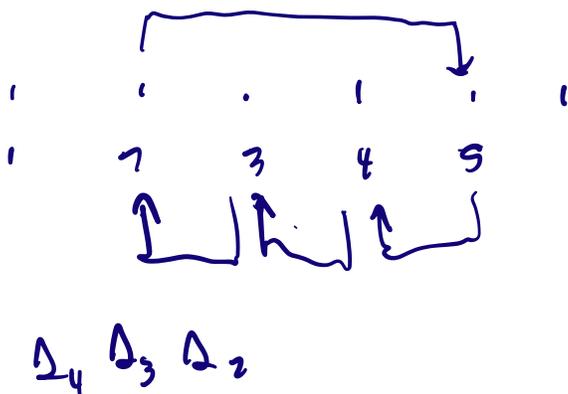


CRYSTAL STRUCTURE ON PIPEDREAMS  
 CLOSELY RELATED TO CRYSTAL STRUCTURE  
 (= MORSE - SCHILLING) ON DECREASING FACTORIZATIONS.

A DECREASING PERMUTATION IS ONE OF  
 THE FORM  $\Delta_{i_1} \dots \Delta_{i_n}$   $i_1 > i_2 > \dots > i_n$ .



IF  $w \in W$

$\phi : w = w_1 \dots w_n$   $w_i$  DECREASING

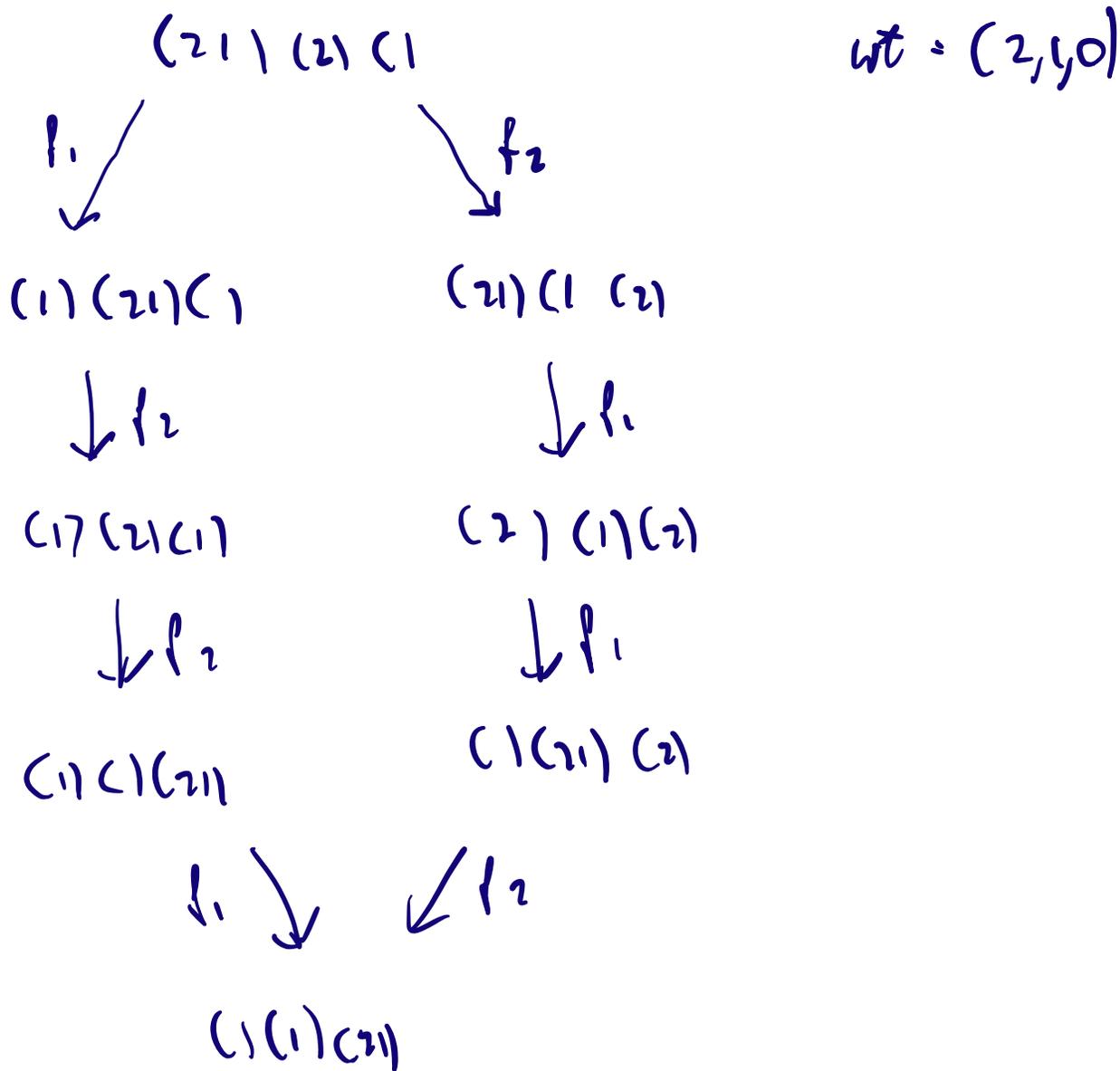
MORSE - SCHILLING :

THERE IS A CRYSTAL STRUCTURE ON  
 DECREASING FACTORIZATION.

$\text{wt} : \text{DECREASING FACT'N} \rightarrow \mathbb{Z}^n$

$\text{wt}(w_1, \dots, w_n) = (l(w_1), l(w_2), \dots)$

$$\Delta_1 \Delta_2 \Delta_1 = \Delta_2 \Delta_1 \Delta_2$$



THIS CRYSTAL  $\mathcal{B}_{\Delta_1 \Delta_2 \Delta_1} \cong \mathcal{B}_{(2,1,0)}^{GL(3)}$

THM: (MORSE-SCHILLING)

$$\mathcal{B}_{w_0} \cong \mathcal{B}_p^{GL(n)}$$

THIS IMPLIES

STANLEY'S THM:

# REDUCED WORDS =

$$\frac{\binom{n}{2}!}{1^{n-2} \cdot 3^{n-4} \dots}$$

STANLEY SYMMETRIC FN

$$= \sum_{\text{RED FACT'N } \phi} z^{\text{wt}(\phi)}$$

RED FACT'N  
 $\phi$ ,

$$\phi = w = w_1 \cdots w_n \quad \text{DECREASING}$$

SUPPOSE WE LOOK AT THE COEFFICIENT

$$\text{OF } z_1 z_2 \cdots z_k$$

$$l(w) = k$$

$$(\Delta_{i_1}) (\Delta_{i_2}) \cdots (\Delta_{i_k}) ( )$$

THIS COEF = # OF FACT'NS,

AND IF  $w = w_0$

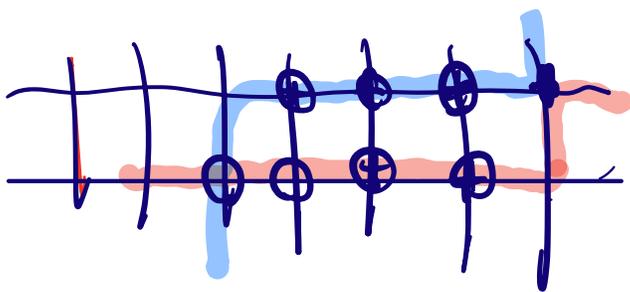
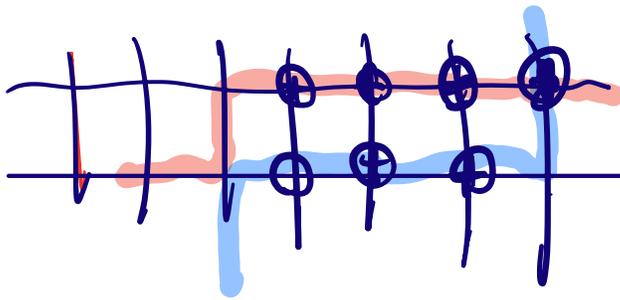
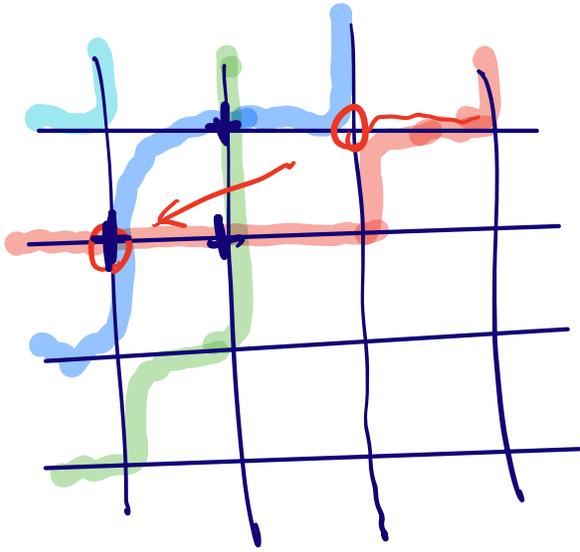
$$\mathbb{B}_{w_0} = \mathbb{B}_\rho$$

# FACT'N = COEF OF  $z_1 \cdots z_k$  IN

$$\mathbb{Q}_\rho$$

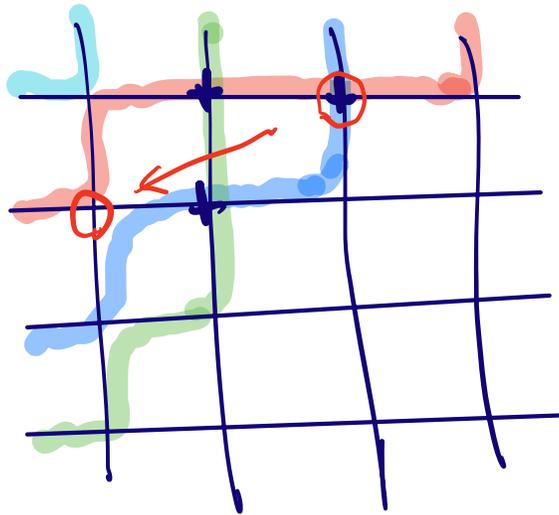
CAN BE COMPUTED.





MINIMAL DISTURBS THE PIPE DREAM.

THE SCUTE MOVE CORRESPONDS TO A  $f_i$   
FOR THE MORSE SCHILLING CRYSTAL.

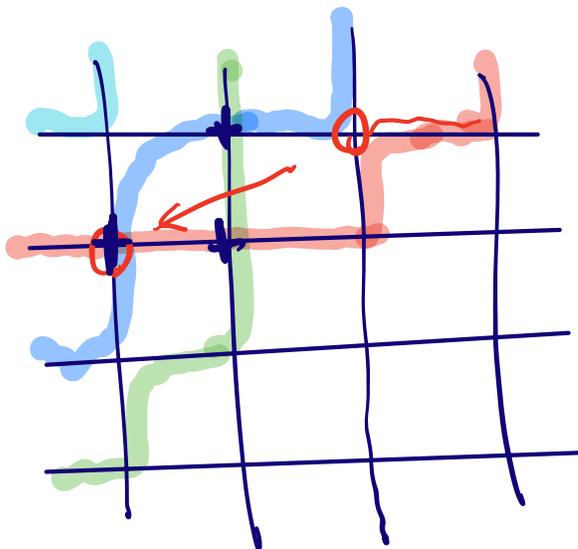


$$(\Delta_3 \Delta_2) (\Delta_3) ( ) ( )$$

IF THERE IS A CROSS AT  $(i, j)$   
WE HAVE A REFLECTION  $\Delta_{i+j-1}$ .

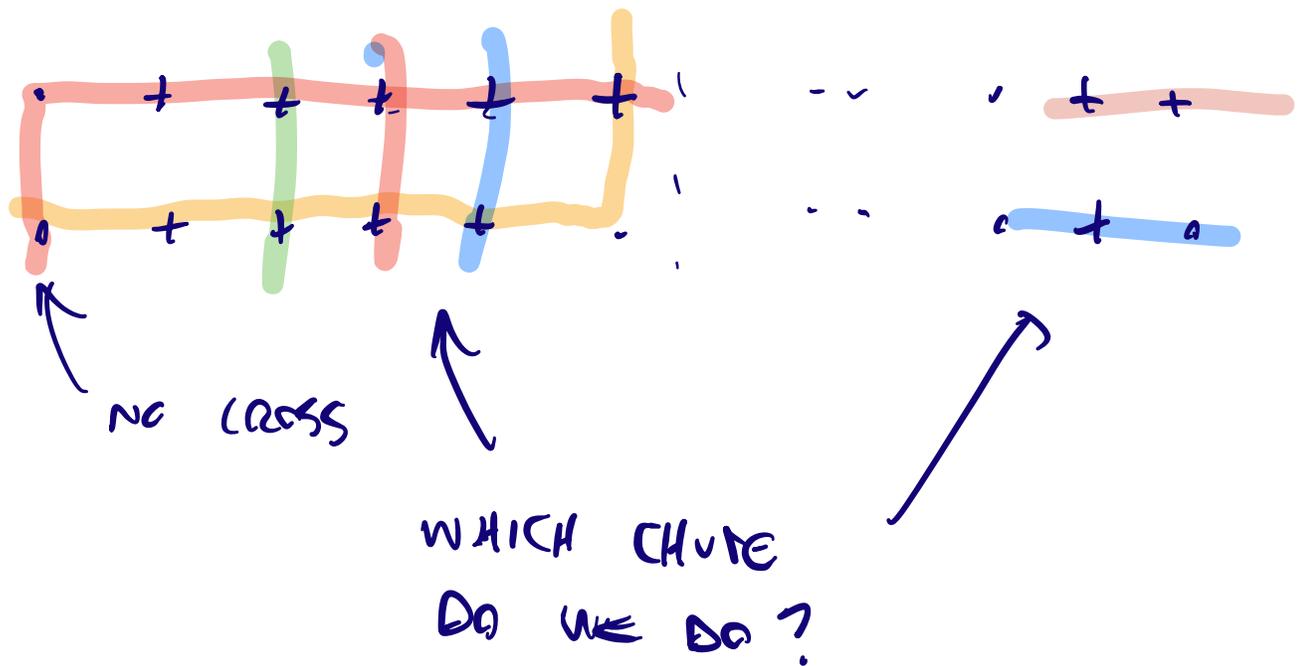
ARRANGE THESE BY ROWS

$$- (\Delta_2) (\Delta_3 \Delta_2) ( ) ( )$$

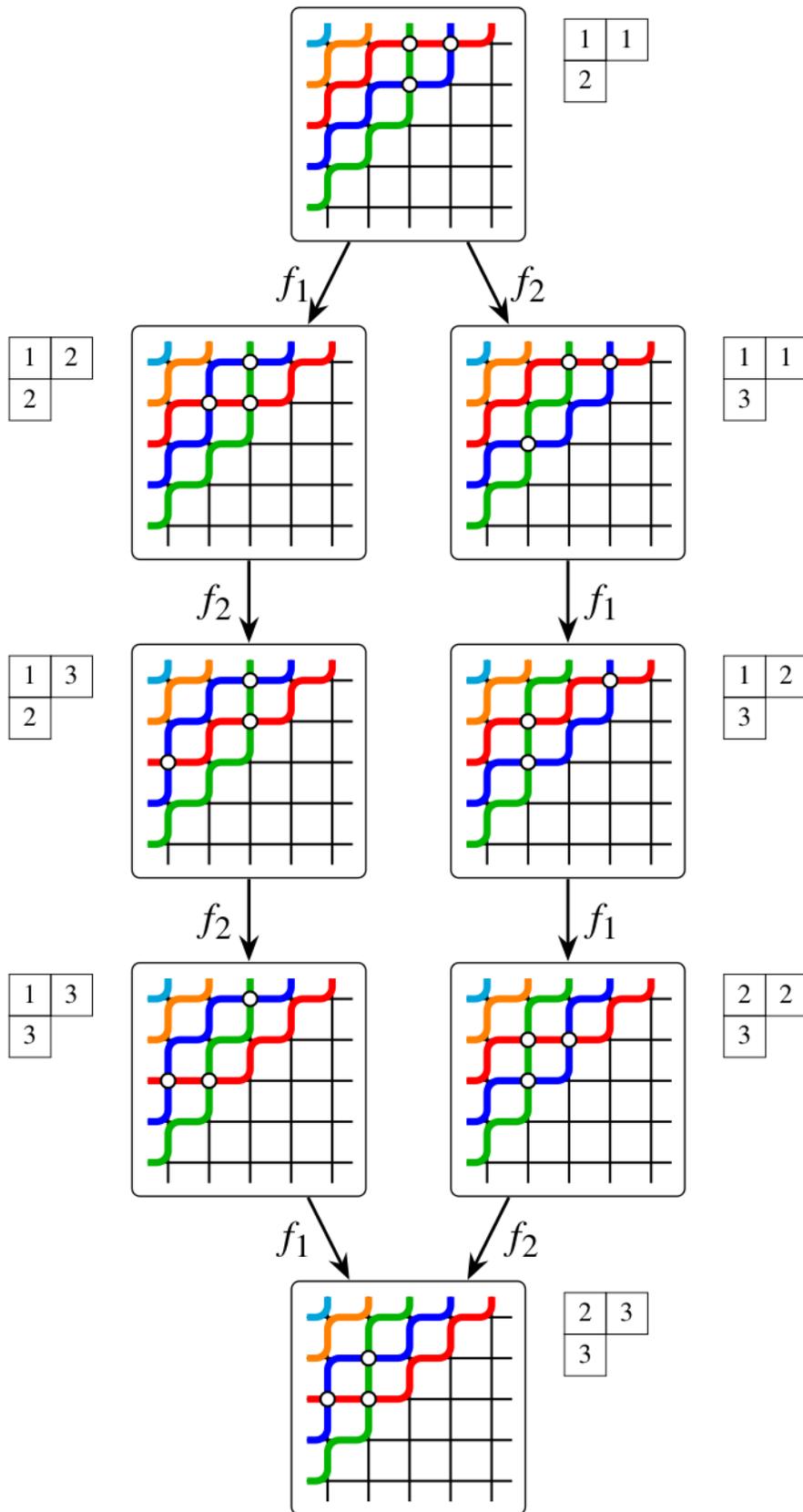


THERE COULD BE  
SEVERAL CHUTE OPERATIONS POSSIBLE  
IN  $n, n+1$  ROWS.

A CHUTE OPERATION IS POSSIBLE  
IF PATTERN OF CROSSINGS IS:



THIS IS ANALOGOUS TO THE PROBLEM  
THAT



THESE ARROWS ARE CHUTE OPERATIONS

$$\omega = 1\ 2\ 5\ 4\ 3 \quad ; \quad \Delta_3 \Delta_4 \Delta_3 = \Delta_4 \Delta_3 \Delta_4$$

THERE IS A DIFFERENCE BETWEEN  
THE MORSE - SCHILLING CRYSTALS  
AND THESE .

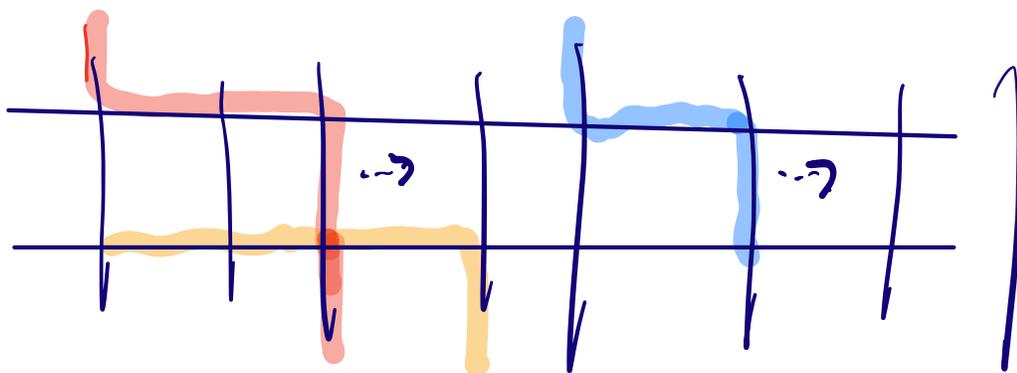
MORSE SCHILLING CRYSTALS PROVIDE  
STANLEY'S SYMMETRIC FUNCTIONS.

PIPEDREAM CRYSTALS MAY BE  
DEMAZURE CRYSTALS.

PROBLEM: HOW TO DECIDE WHICH  
CRYSTAL OPERATION  $f_i$  DOES.

SIMILAR TO: IN TAKUYAMA  $q=0$   
MODELS  $f_i$  MOVES ONE "LEG"...

TOKUYAMA,



$P_i$  MIGHT MOVE RED

OR IT MIGHT MOVE BLUE

DECIDING WHICH ONE IS MOVED IS  
ANALOGOUS TO DECIDING WHICH CHORE  
OPERATION IS PERFORMED

ON FRIDAY I'LL ANSWER BOTH QUESTION.  
WE HAVE TO PERFORM A "BRACKETING"  
CALCULATION TO DECIDE WHICH IS MOVED.