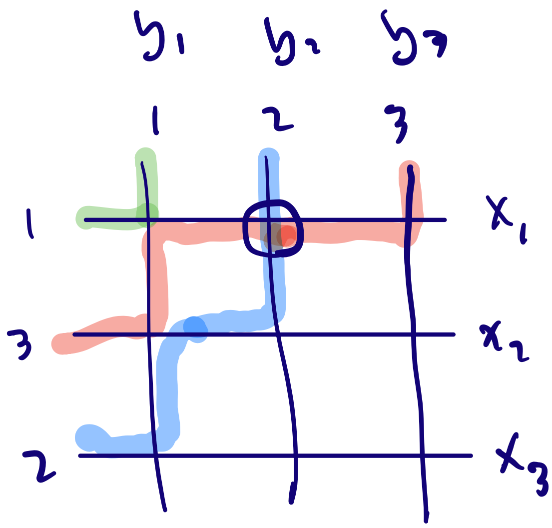
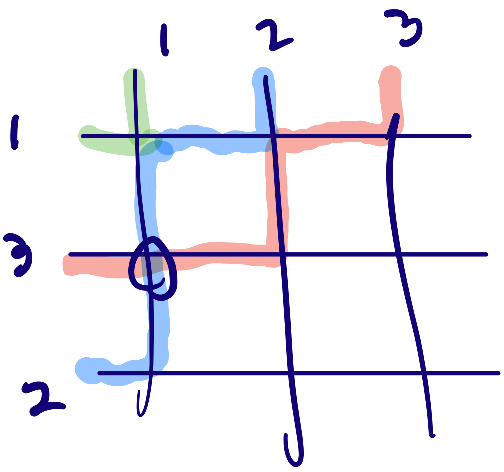
REVIEW: CLASSICAL PIPEDREAMS:



$x_1 - y_2$

CROSSING AT (i, j) LOCATION CONTRIBUTES

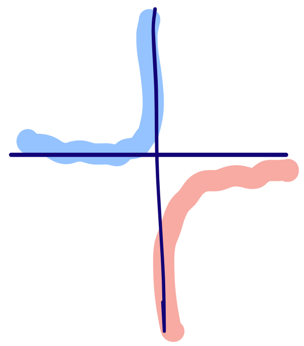
$x_i - y_j$



$x_2 - y_1$

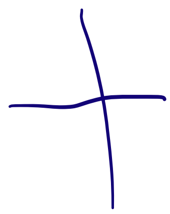
PARTITION FUNCTION $x_1 - y_2 + x_2 - y_1$

BUMPLESS:



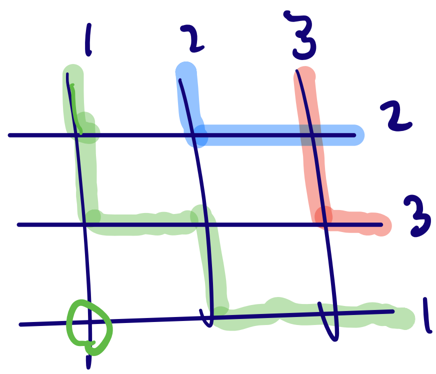
"BUMP" NOT ALLOWED

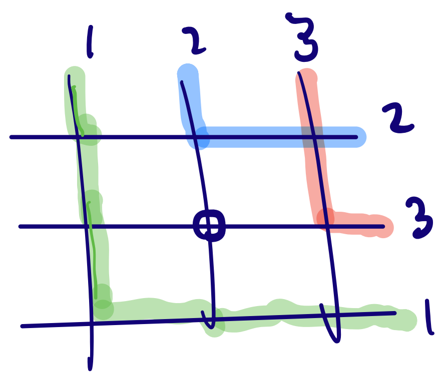
CONTRIBUTION IS FROM EMPTY STATES:



$x_i - y_j$ CONTRIBUTION.

ROWS ARE NUMBERED BOTTOM TO TOP.





$x_1 - y_1$

$x_2 - y_2$

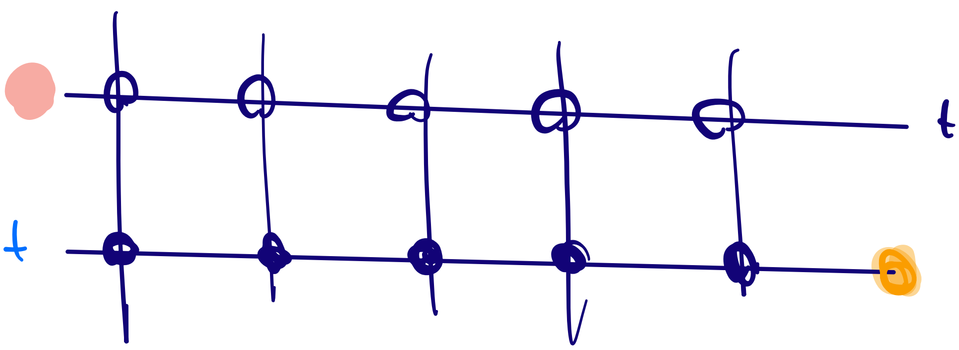
$$x_1 - y_1 + x_2 - y_2$$

SAME AS BEFORE: THE FACT THE PARTITION FUNCTIONS ARE EQUAL DOES NOT FOLLOW FROM A STATE-WISE BIJECTION.

IF $y=0$ HUANG-GAO PRODUCED A BIJECTION BETWEEN CLASSICAL AND BUMPLESS PIPEDREAMS

THERE ARE 2 THINGS WE CAN DO;

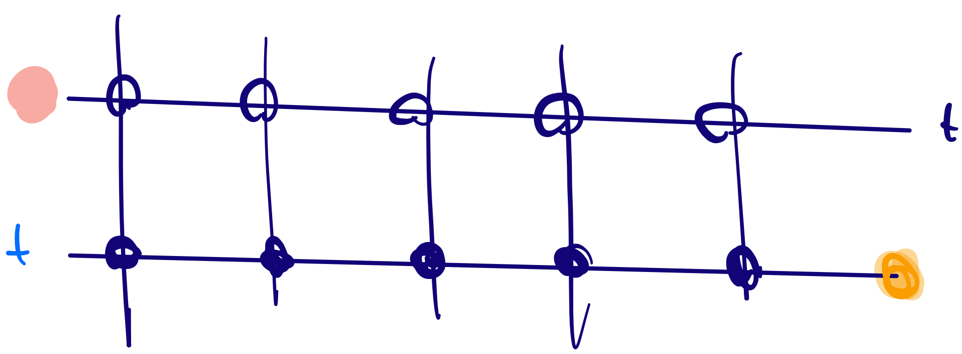
(1) INTERCHANGE A ROW OF CLASSICAL WITH A ROW OF BUMPLESS.



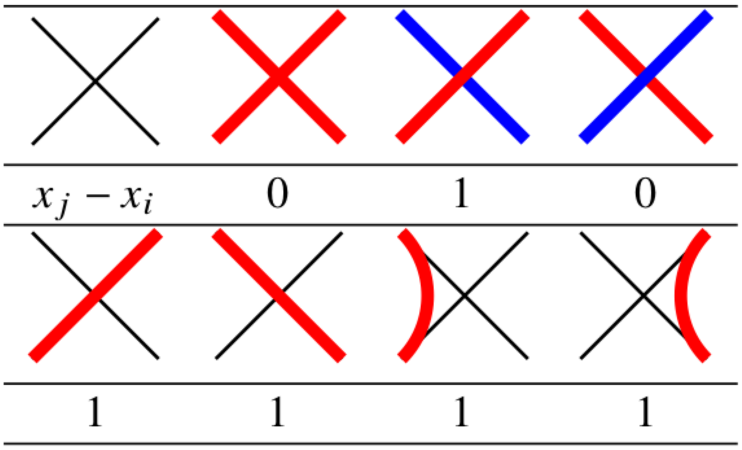
CLASSIC

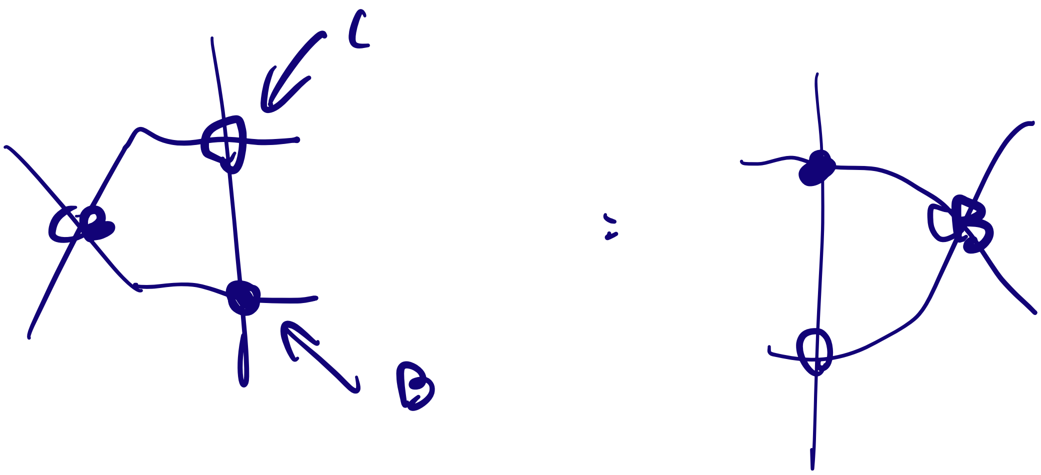
BUMPLESS.

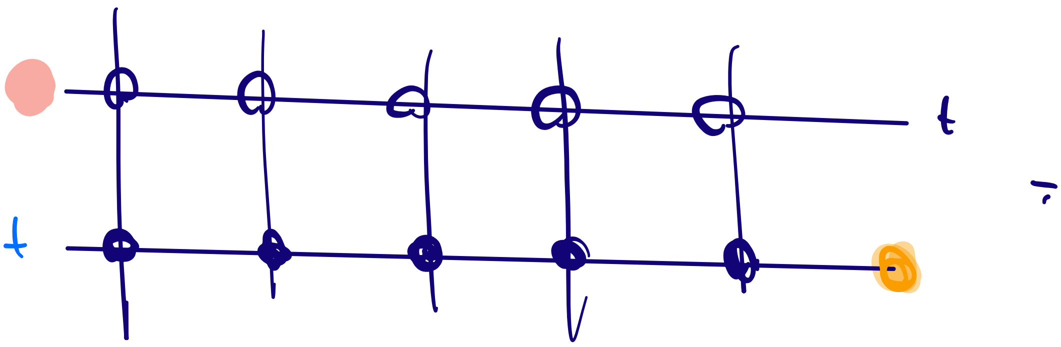
=

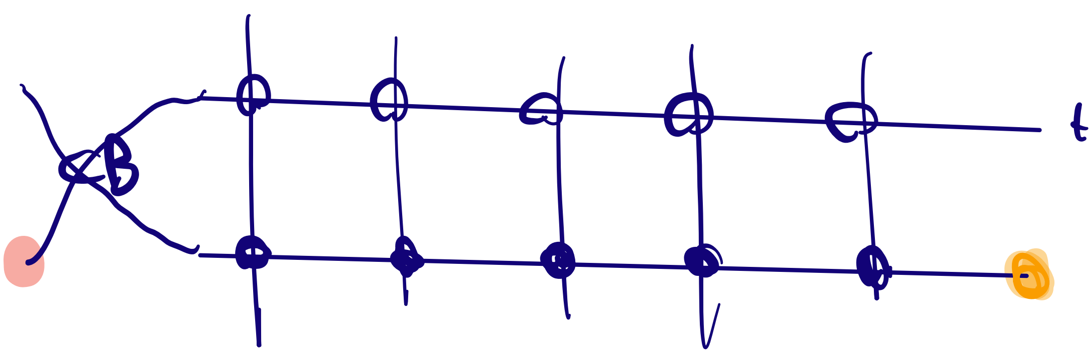


THIS R-MATRIX WORKS WITH CLASSIC & BLESS





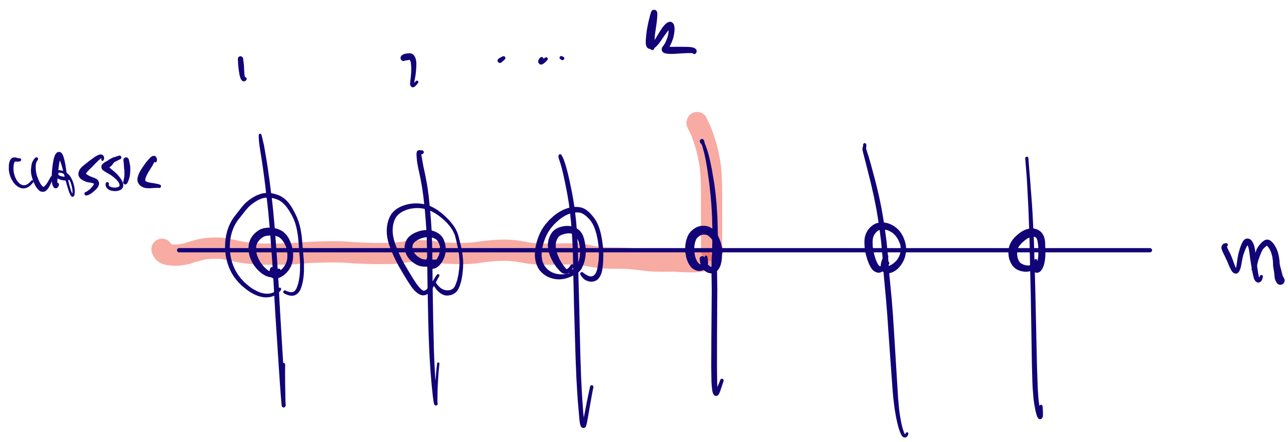




= YBE & DETACH.

THIS IS DONE WITHOUT YBE BY KNUTSON – UDELL.

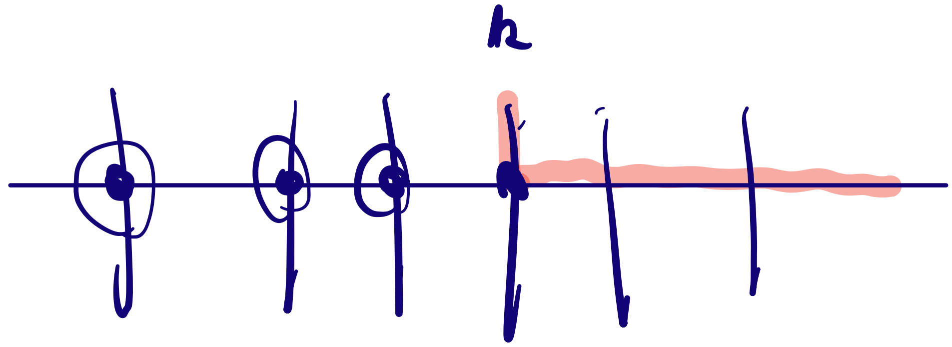
SECOND OPERATION THAT DOES NOT CHANGE THE PARTITION FUNCTION:



IF THE DESCENT INTO THE ROW OF CLASSIC PIPE IS IN COLUMN k, CONTRIBUTES

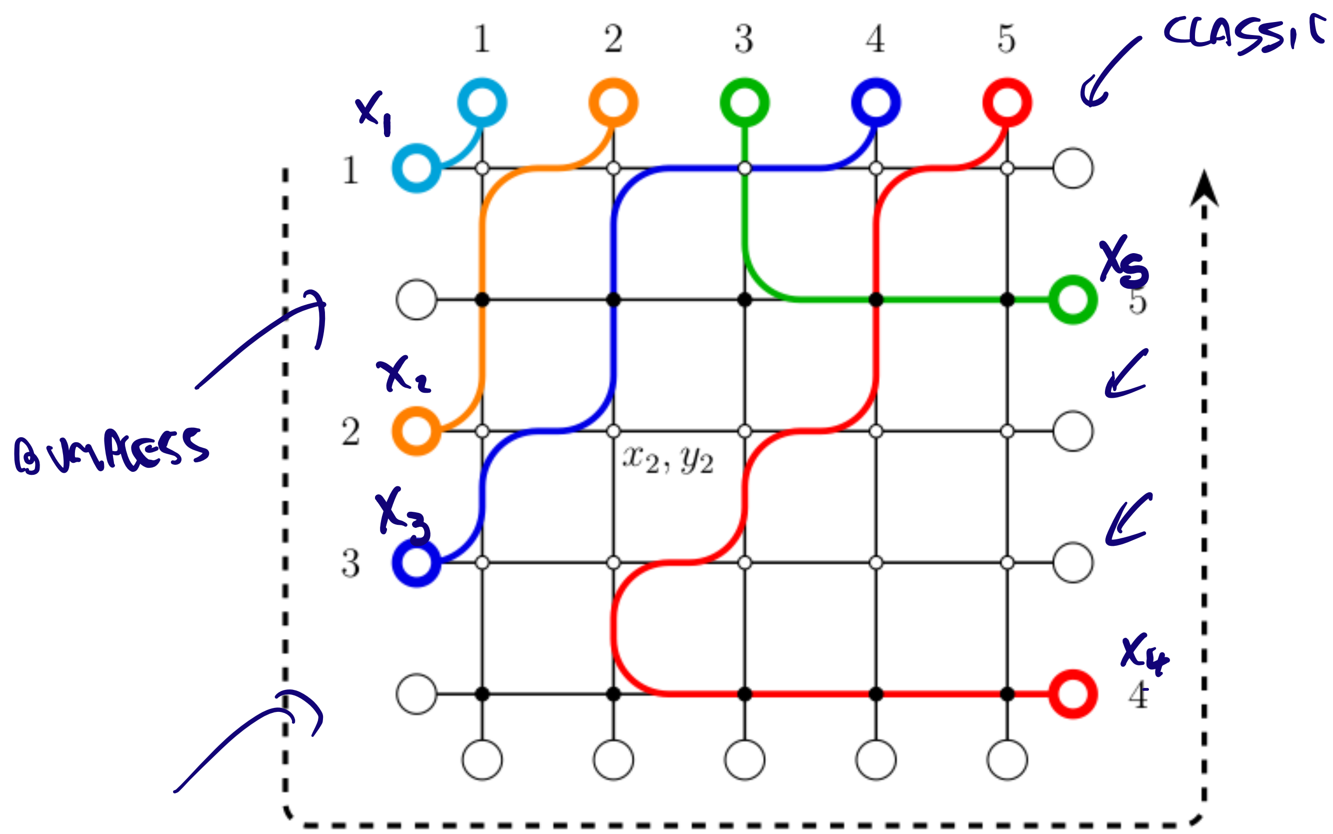
$$\prod_{j=1}^{k-1} x_m - y_j$$

BUMPIES =



HYBRID MODELS:

CHOOSE A SUBSET OF ROWS TO BE CLASSIC. THE REST WILL BE BUMPLESS.



LABEL THE ROWS IN ORDER SHOWN BY ARROW.

COLUMNS $1, \dots, n$ AS USUAL.

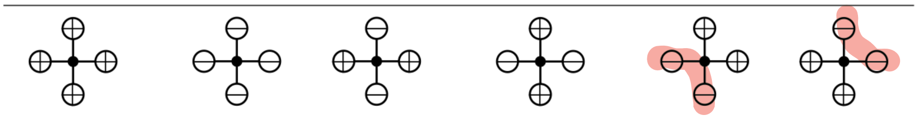
THE PARTITION FUNCTION OF ALL THESE MODELS ARE EQUAL.

CLEAR FROM OPERATIONS ALLOWED

(SWITCH B & C ROWS OR CHANGE LAST ROW $B \leftrightarrow C$.)

THIS IS SIMILAR TO TOKUYAMA GAMMA AND DELTA MODELS:

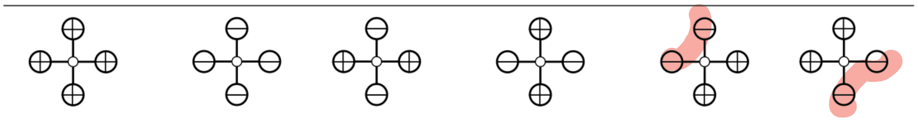
Γ



$\mathtt{a}_1$	$\mathtt{a}_2$	$\mathtt{b}_1$	$\mathtt{b}_2$	$\mathtt{c}_1$	$\mathtt{c}_2$
1	z_i	0	$z_i + \alpha_j$	z_i	1

The Boltzmann weights for the *Delta model* are given as follows.

Δ



$\mathtt{a}_1$	$\mathtt{a}_2$	$\mathtt{b}_1$	$\mathtt{b}_2$	$\mathtt{d}_1$	$\mathtt{d}_2$
$z_i + \alpha_j$	1	$-\alpha_j$	1	1	z_i

GAMMA OR BUMPLESS : RIGHTMOVING

DELTA : LEFT MOVING.

WE CAN SIMILARLY SWITCH A GAMMA AND A DELTA LAYER USE YBE.

AND WE CAN CHANGE BOTTOM ROW FROM Γ TO Δ.

BOTH GAMMA AND DELTA MODELS HAVE SAME PARTITION FUNCTION $Z_\lambda(z)$ AND A DIRECT PROOF OF EQUALITY INVOLVES HYBRID MODELS.

(CH. 4 OF NOTES.)

GAMMA AND DELTA ICE OCCUR TOGETHER IN VARIOUS DIFFERENT CONTEXTS.

ONE SETUP IS DESCRIBED IN CHAPTER 13

BOTH GAMMA AND DELTA ICE MAY BE EXTRAPOLATED TO INFINITE GRIDS.

	a_1	a_2	b_1	b_2	c_1	c_2
Δ-ice	$\begin{smallmatrix} & \oplus & \\ \oplus & z,q & \oplus \\ & \oplus & \end{smallmatrix}$	$\begin{smallmatrix} & \ominus & \\ \ominus & z,q & \ominus \\ & \ominus & \end{smallmatrix}$	$\begin{smallmatrix} & \ominus & \\ \oplus & z,q & \oplus \\ & \ominus & \end{smallmatrix}$	$\begin{smallmatrix} & \oplus & \\ \ominus & z,q & \ominus \\ & \oplus & \end{smallmatrix}$	$\begin{smallmatrix} & \oplus & \\ \ominus & z,q & \oplus \\ & \ominus & \end{smallmatrix}$	$\begin{smallmatrix} & \ominus & \\ \oplus & z,q & \ominus \\ & \oplus & \end{smallmatrix}$
	1	$-qz$	1	z	$(1-q)z$	1

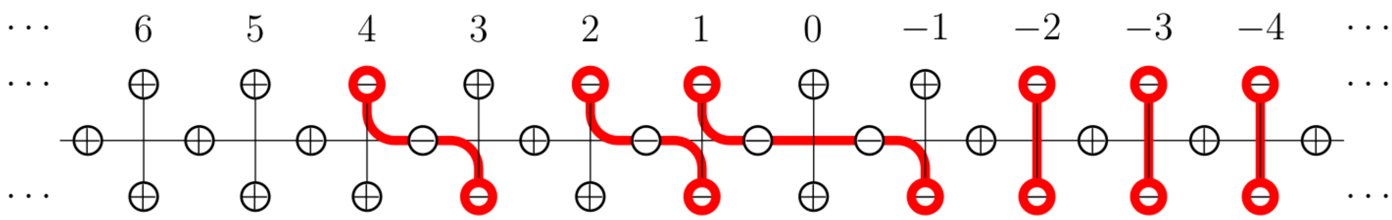
TO COMPARE WITH ABOVE $q=0$ AND

HERE I AM SUPPRESSING α_i THOUGH THOSE COLUMN PARAMETERS CAN BE INCLUDED. ALSO I'M REVERSING SIGNS ON HORIZONTAL EDGES.

$$\begin{array}{ccc} & + & \\ + & & + \\ & + & \end{array} \qquad\qquad \begin{array}{ccc} & - & \\ + & & + \\ & - & \end{array}$$

HAVE WEIGHT 1.

CONSIDER AN INFINITE GRID.



ALL PLUS

MIXTURE,

ALL −

ALL BUT FINITELY MANY VERTICES ARE

α, OR β, SO THE PARTITION FUNCTION IS AN INFINITE PRODUCT BUT ONLY FINITELY MANY FACTORS ARE 1.

ON FRIDAY I'LL DISCUSS GAMMA AND DELTA ICE IN THIS SETUP.

CORRESPONDS TO CHAPTER 13 IN NOTES,

AFTER THIS I'LL DISCUSS CRYSTALS FOR PIPEDREAMS.