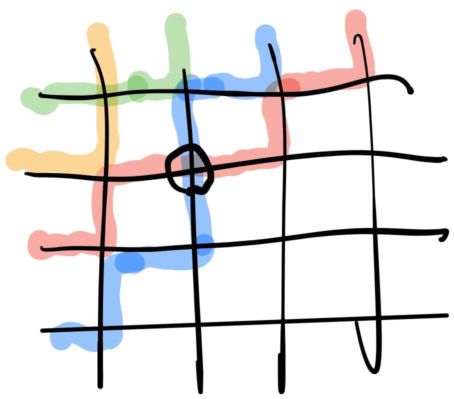
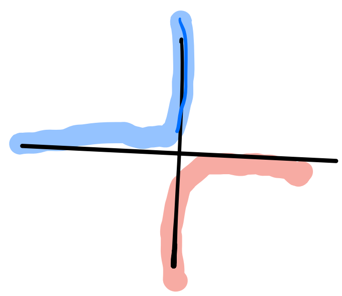
BUMPLESS PIPE DREAMS.

TWO DIFFERENT MODELS FOR DOUBLE SCHUBERT POLYNOMIALS. BUMPLESS PIPE DREAMS WERE INTRODUCED BY LEE, LAM, SHIMOZONO.

CLASSIC:

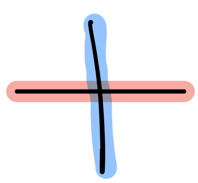




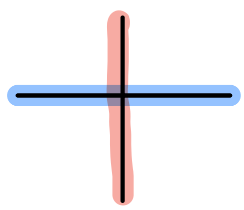
"BUMP"

FORBIDDEN IN THE BUMPLESS MODELS.

FOR CLASSIC PIPEDREAMS 



ALLOWED



FORBIDDEN

CROSSING CONVENTION IS REVERSED IN BPD.

$x_i - y_j$	1	1	1	1	1

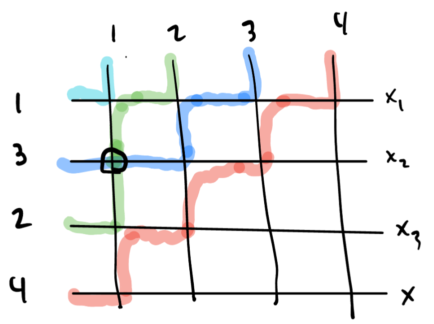
BPD WEIGHTS.

FOR CLASSIC WE GET A CONTRIBUTION IF THERE IS A CROSSING

$$x_i - y_j \quad \text{in } x_i \text{ ROW } j\text{-th COL.}$$

FOR BPD CONTRIBUTION IF NO PATHS

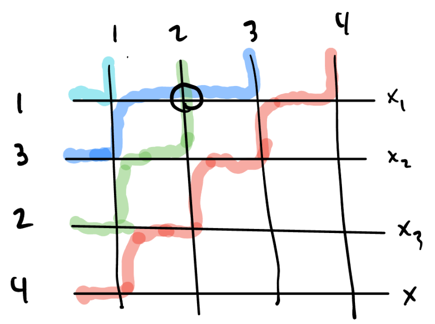
FOR CLASSIC ROWS ARE LABELED $x_1, \dots, x_n$ TOP TO BOTTOM. FOR COLS $x_1, \dots, x_n$ BOTTOM TO TOP.



$x_2 - y_1$

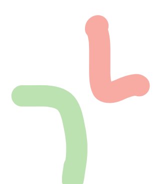
$$S_{(1324)} = S_{s_2}$$

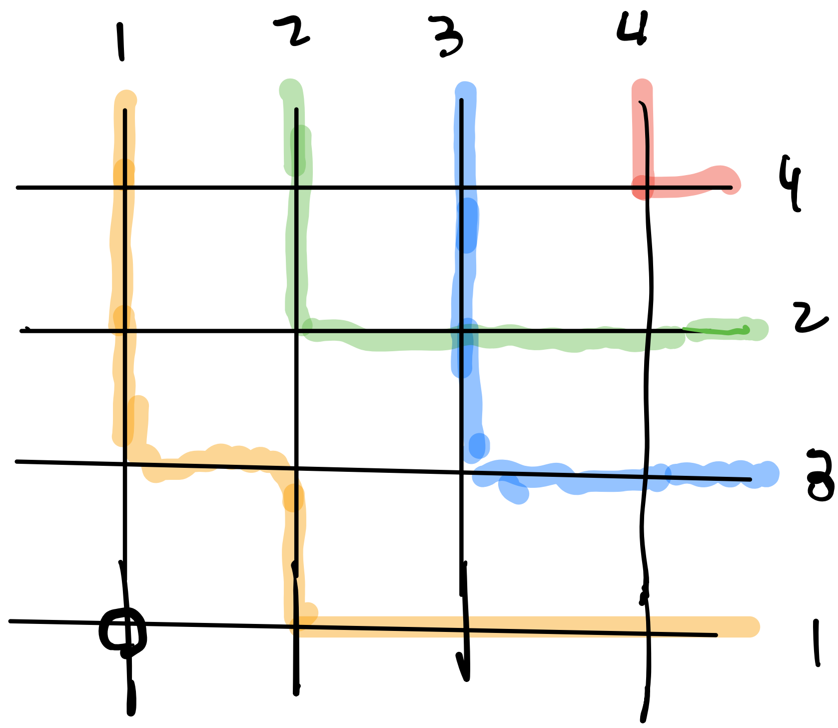
$$x_2 - y_1 + x_1 - y_2.$$



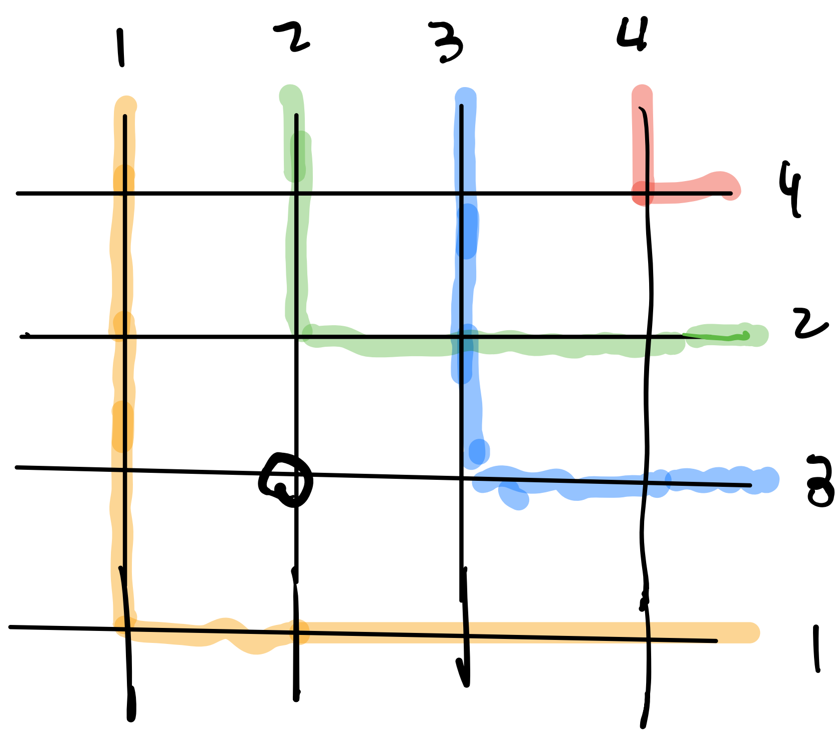
$x_1 - y_2$

CLASSIC PD.





$x_1 - y_1$



$x_2 - y_2$

DOUBLE SCHUBERT =

$$\mathfrak{S}_{(1324)} = x_1 - y_1 + x_2 - y_2$$

WE GET SAME ANSWER.

DEF: $$S_{w_0}(x;y) = \prod_{i+j\leq n} (x_i - y_j)$$

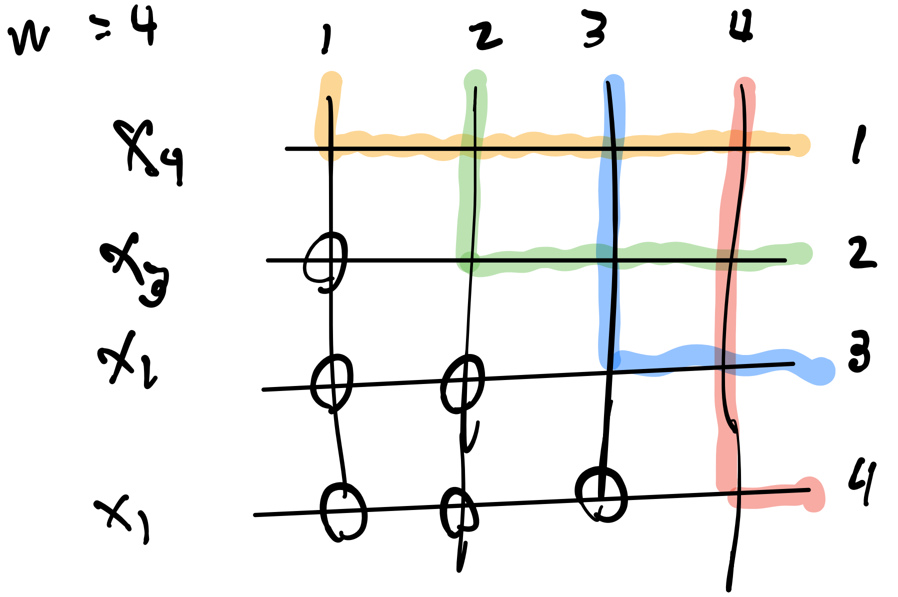
IF $w\Delta_i < w$

$$S_{w\Delta_i} = D_i S_w$$

$$D_i f(x) = \frac{f(x) - f(\Delta_i x)}{x_i - x_{i+1}}$$

APPLY THE OPERATOR TO X VARIABLES.

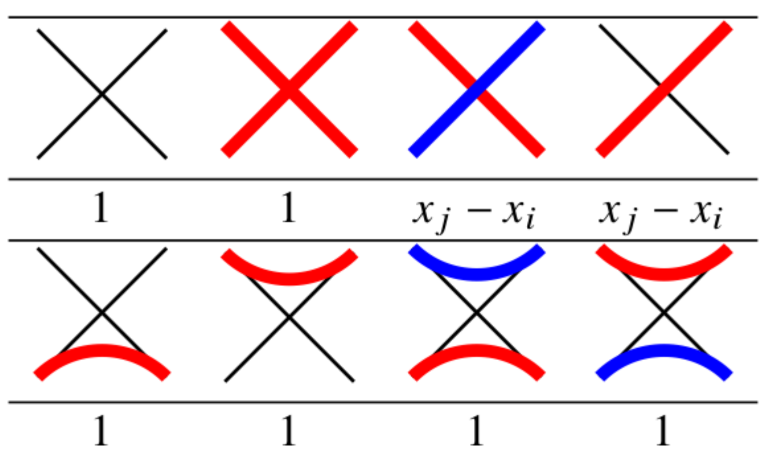
EVALUATION FOR w_0 IS STRAIGHTFORWARD.



$w_0 = 4321$

$(x_1 - y_1)(x_2 - y_1)(x_3 - y_1)$
$(x_1 - y_2)\ (x_2 - y_2)$
$(x_1 - y_3)$

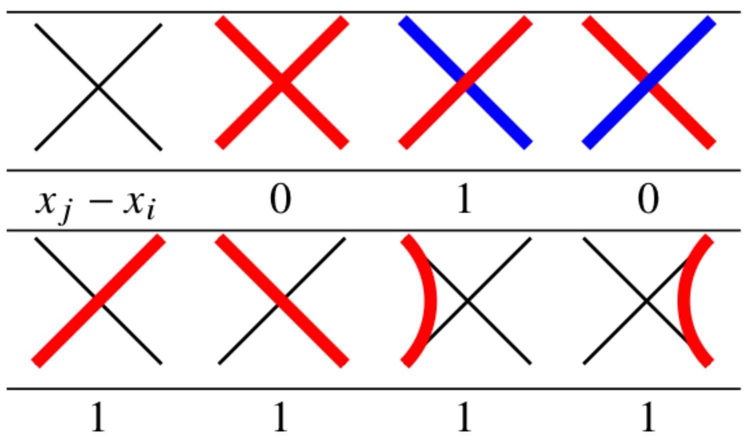
YOU CAN PROVE IF $w s_i < w$
THEN $\quad \mathfrak{S}_{w s_i} = \partial_i \mathfrak{S}_w$
CAN BE PROVED USING YBE.



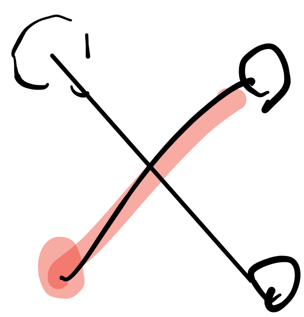
R-MATRIX
FOR BUMPLESS
PIPES.

SIMILAR TO LECTURE 13,
KNUTSON AND UDELL (FPSAC PAPER)
CONSIDERED HYBRID MODELS WITH LAYERS
OF CLASSIC AND BUMPLESS PIPE

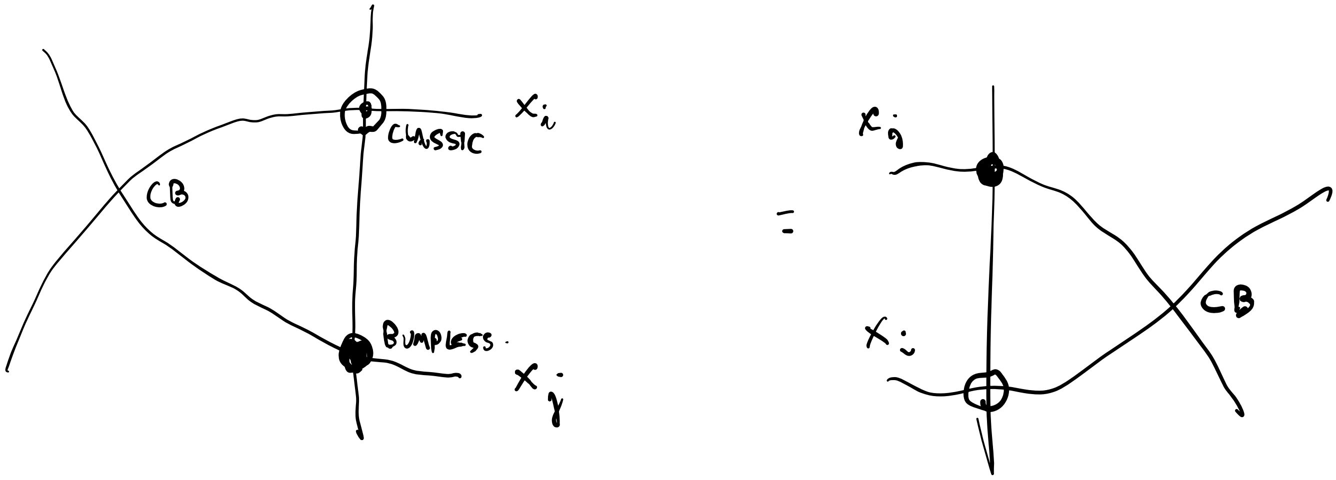
INSTEAD OF KV THEOREM & UDELL USE
ANOTHER YANG-BAXTER EQUATION.



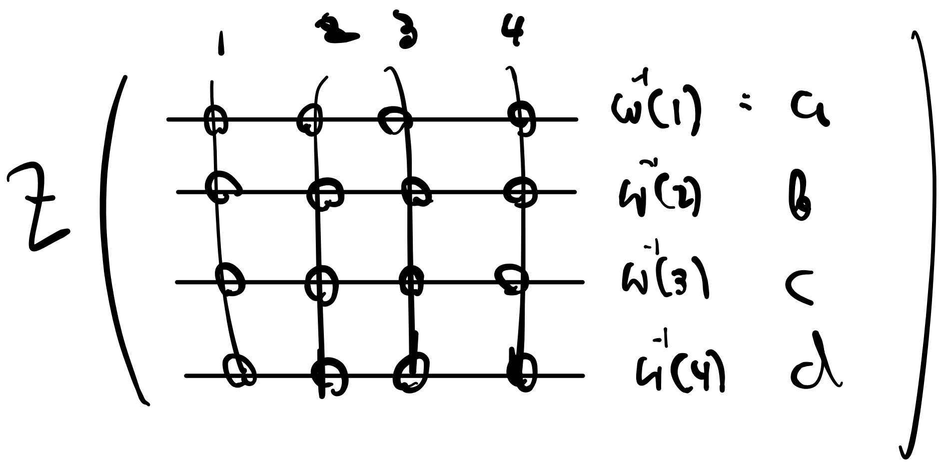
CB R-MATRIX.



OTHER HYBRID MODELS OF
KNUTSON - PZJ (PAUL ZINN-JUSTIN)

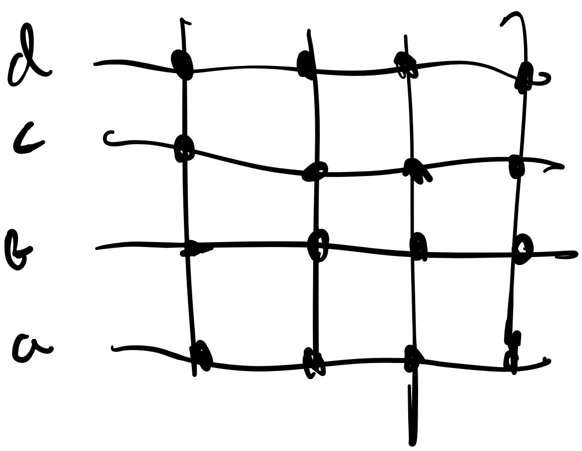


WE CAN USE THIS AS FOLLOWS



ALL CLASSIC

SERIES OF HYBRID MODELS.



ALL BUMPLESS

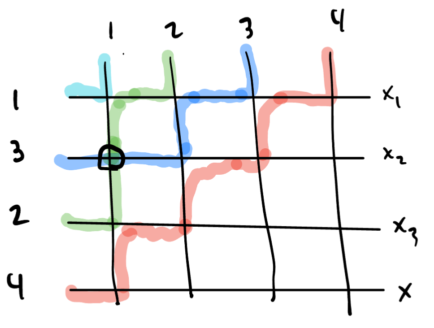
STATES

$$\mathfrak{S}_w((1,\dots,1);(0,\dots,0))$$

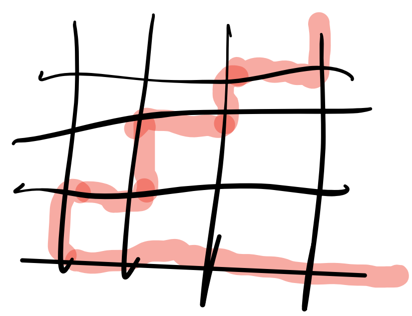
EXPLICIT.
BIJECTION BETWEEN STATES
FOUND BY HUANG, GAO.
IF y VARIABLES ARE ZERO.
THE STATES HAVE SAME VALUE.

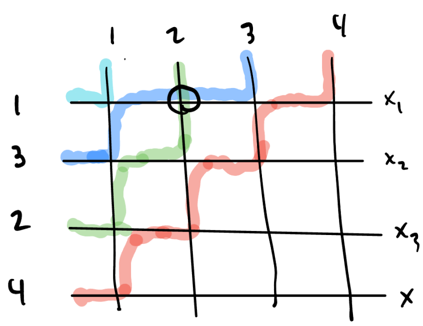
BIJECTIVE APPROACH WON'T WORK
FOR DOUBLE SCHUBERTS.

HOW IT WORKS:
AT BOTTOM WE CAN SIMPLY CHANGE
THE STATE FROM CLASSIC TO BUMPLESS



$x_2 - y_1$



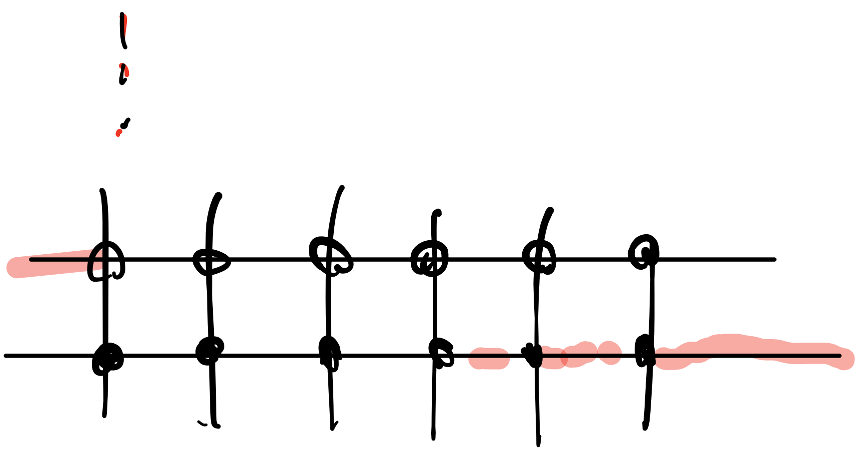


$x_1 - y_2$

THERE
CAN BE
NO CROSSING.
ON BOTTOM

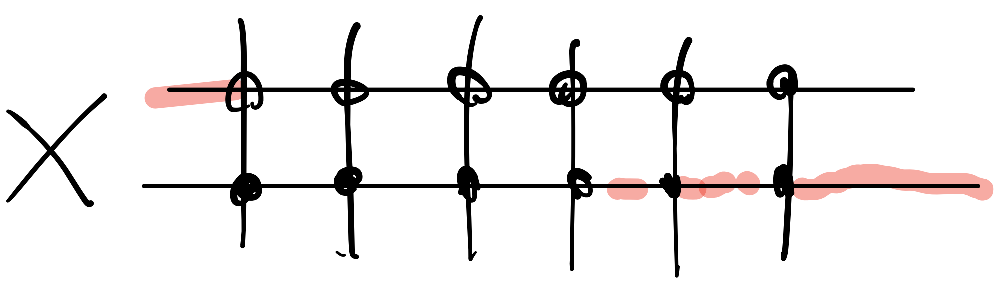
NO 
EMPTY
ON BOTTOM.

THIS BRUTE FORCE CHANGE OF BOTTOM ROW FROM CLASSIC TO BUMPLESS DOESN'T CHANGE THE WEIGHT OF STATE.



BOTTOM.

ATTACH R-MATRIX



BOTTOM

TO BE CONTINUED.