WE WANT TO USE YBE TO PROVE THAT CLASSIC PIPEDREAMS DO REPRESENT SCHUBERT POLYNOMIALS. MOREOVER THEY IMMEDIATELY SHOW US HOW TO INTRODUCE A SECOND SET OF COLUMN PARAMETERS PRODUCING DOUBLE SCHUBERT (MACDONALD) WHICH REPRESENT EQUIVARIANT COHOMOLOGY.

$$\left\{\begin{pmatrix} * & & \\ & \ddots & \\ & & * \end{pmatrix}\right\} = T \quad G \quad GL_n/B = X$$

SCHUBERT POLYNOMIALS:

$$S_{w_0} = -P = x_1^{n-1} x_2^{n-2} \cdots x_{n-1}$$

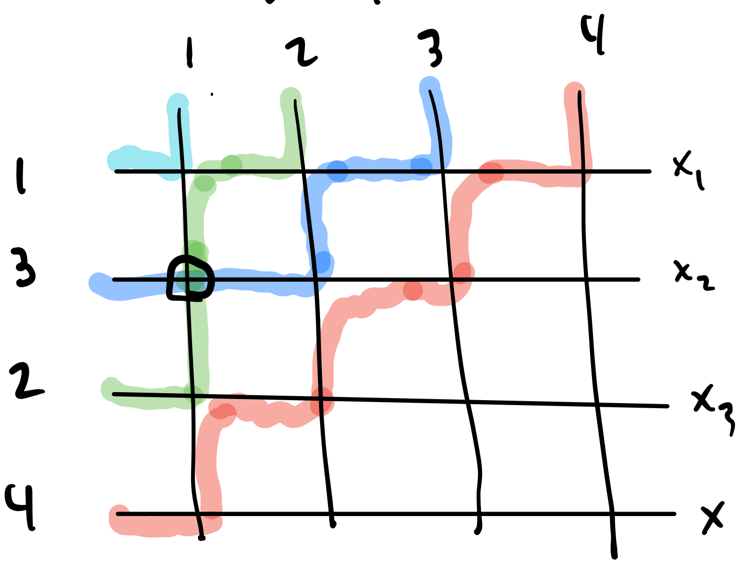
IF $w\sigma_i < w$ THEN

$$S_{w\sigma_i} = D_i S_w \qquad (D_i f) = \frac{f(x) - f(\sigma_i x)}{x_i - x_{i+1}}$$

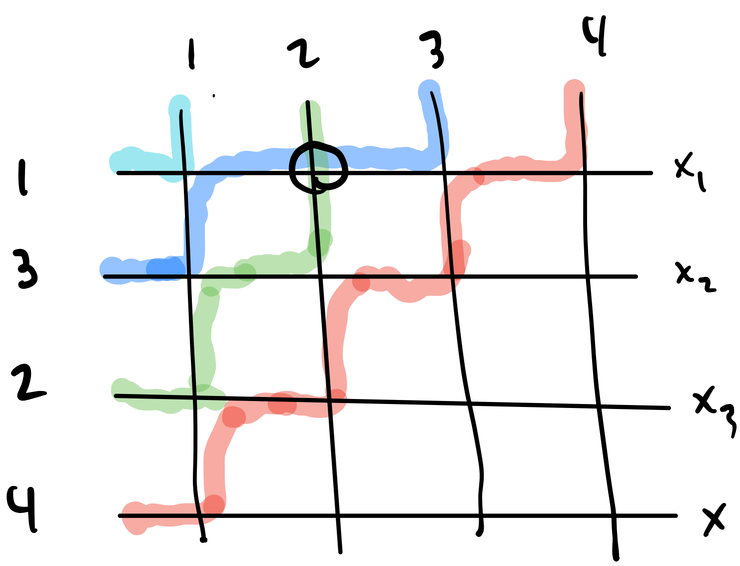
D_i SATISFY BRAID REL'S AND $D_i^2 = 0$

NILHECKE ALGEBRA.

EXAMPLE:



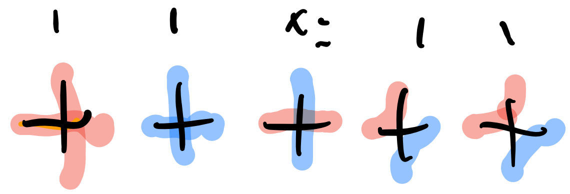
X_2



X_1

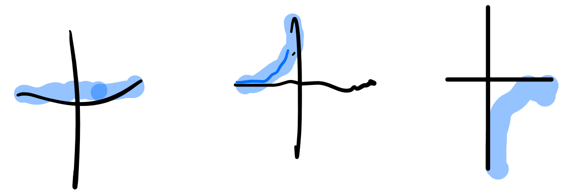
1324 $x_1 + x_2$

Δ_2



R > B

⊕ IS JUST ANOTHER COLOR (SMALLEST)



ALLOWED THOUGH IT CANNOT APPEAR IN PRACTICE

THEOREM: $Z\left(\text{PIPEDREAM MODEL}\right) = S_w$.

IF $w = w_0$ EVERY PAIR OF PATHS MUST CROSS, EASY TO CHECK.

ASSUME $Z(\mathfrak{S}_w) = \mathfrak{S}_w$

AND $w\Delta_i < w$. I WANT TO SHOW HOW YANG-BAXTER EQUATION PROVES

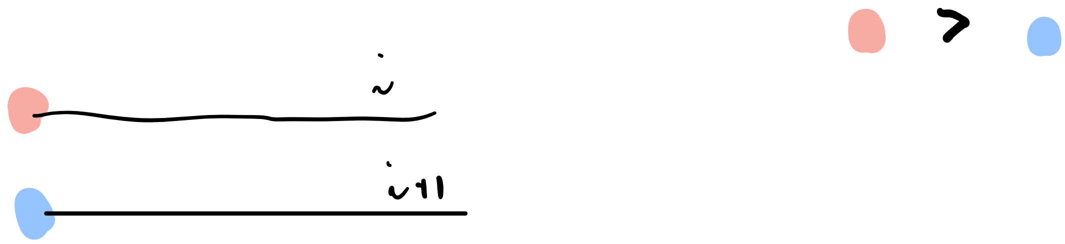
$$Z(\mathfrak{S}_{w\Delta_i}) = \mathfrak{S}_{w\Delta_i} = \partial_i \mathfrak{S}_w .$$

THE CONDITION $w\Delta_i < w$ MEANS $w(i) > w(i+1)$. IF THIS IS TRUE $w\Delta_i$ HAS ONE LESS INVERSION THAN w.

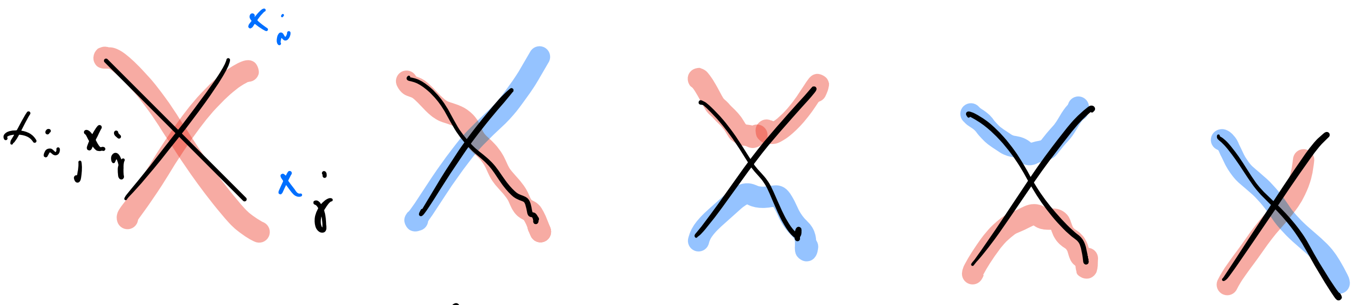
INVERSION: PAIR (i,j) $i < j$ $w(i) > w(j)$

$\ell(w)$ = # OF INVERSIONS.

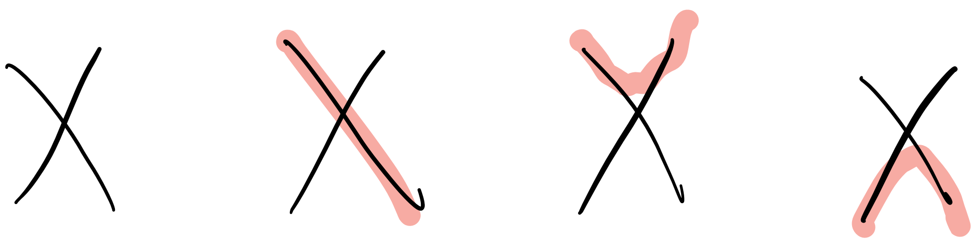
$(i, i+1)$ IS AN INVERSION FOR w BUT NOT $w\Delta_i$.



YANG-BAXTER EQUATION FOR THIS PROBLEM:

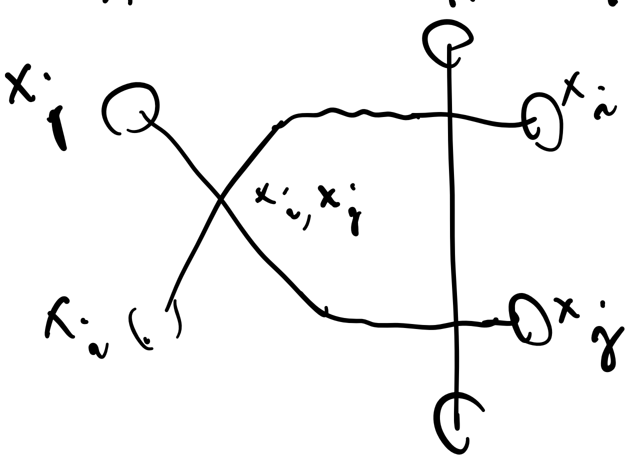


1 $\qquad$ $x_j - x_i$ $\qquad$ 1 $\qquad$ 1 $\qquad$ 0



1 $\qquad$ $x_j - x_i$ $\qquad$ 1 $\qquad$ 1

THIS IS THE R-MATRIX.



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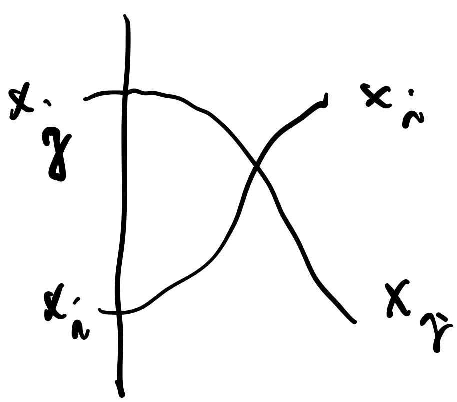
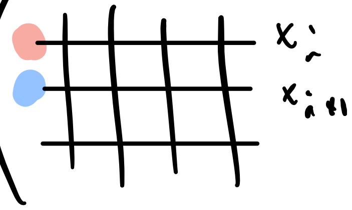
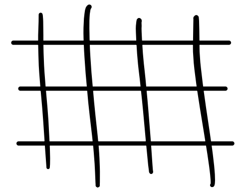
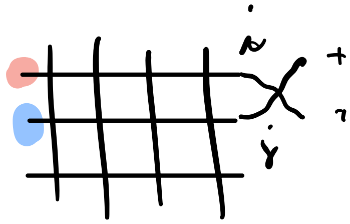
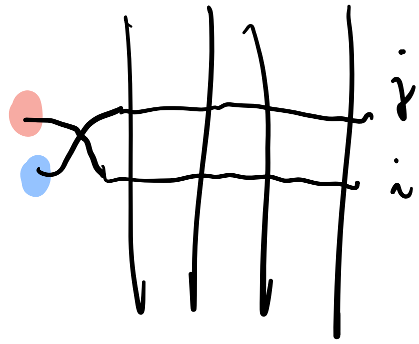
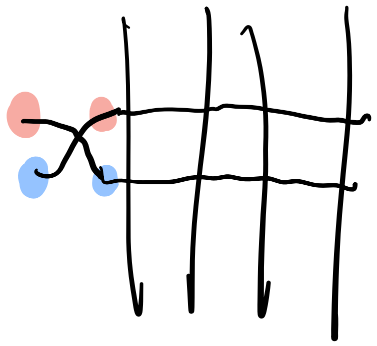
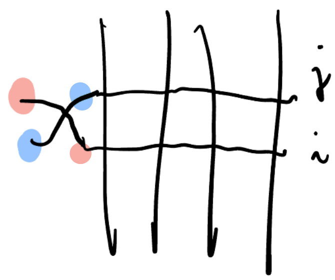


$Z(\omega; x) = Z\Bigg($  $\Bigg)$



$= Z\Bigg($  $\Bigg)$

$= Z\Bigg($  $\Bigg)$

$= Z\Bigg($  $\Bigg) + Z\Bigg($  $\Bigg)$

$$= Z(\omega; \Delta_i x) + (x_i - x_{i+1})\, Z(\omega \Delta_i; \Delta_i x)$$

REPLACE x BY $\Delta_i x$

$$Z(w; \Delta_i x) = Z(w; x) + (x_{i+1} - x_i) Z(w\Delta_i; x)$$

$$Z(w\Delta_i; x) = \frac{Z(w; x) - Z(w; \Delta_i x)}{x_i - x_{i+1}}$$

$$Z(w\Delta_i; x) = D_i Z(w; x)$$

BY DOWNWARD INDUCTION WRT BRUHAT ORDER WE CAN INFER

$$S(w; x) = Z$$

PIPEDREAMS

DOUBLE SCHUBERT POLYNOMIALS.

MACDONALD (MOTIVATED MAYBE BY FACTORIAL SCHUR FUNCTIONS)

SUGGESTED INTRODUCING A SECOND SET OF PARAMETERS

$$\Delta(x;y) = \prod_{i+j\leq n} (x_i - y_j)$$

$S_w(x;y)$ IS DEFINED BY

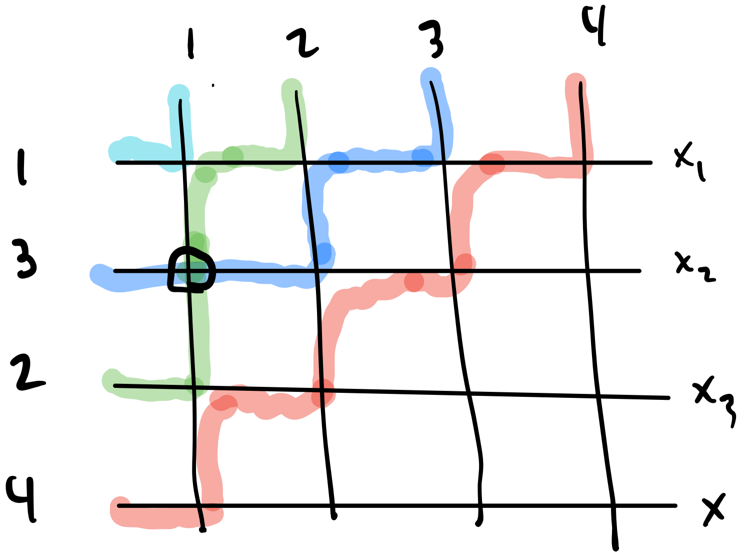
$$S_{w_0}(x;y) = \Delta(x;y)$$

AND IF $w s_i < w$ THEN

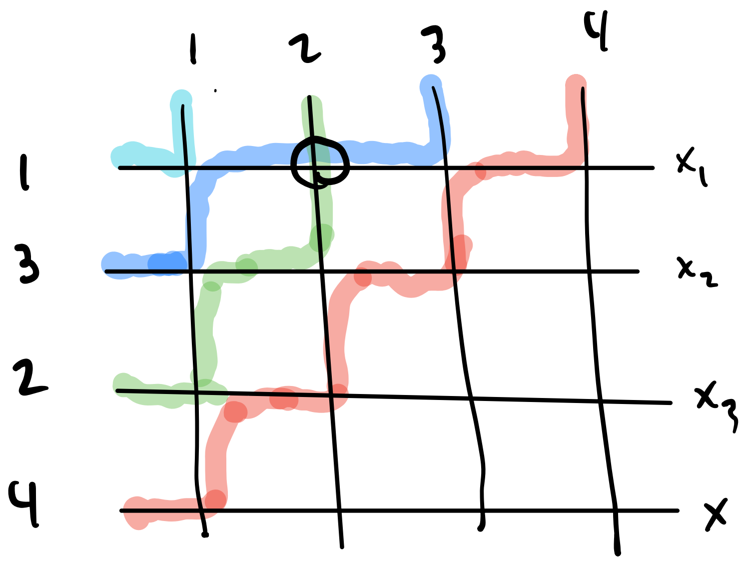
$$S_{ws_i}(x;y) = \partial_i S(x;y).$$

WE CAN MODIFY PIPEDREAMS BY GIVING A CROSSING IN i-ROW AND j-COLUMN WEIGHT $x_i - y_j$.

Example:



$$x_2 - y_1$$



$$x_1 - y_2$$

$$S(\Delta_2; x, y) = x_2 - y_1 + x_1 - y_2.$$

PROOF IS IDENTICAL SINCE YBE IS STILL VALID.