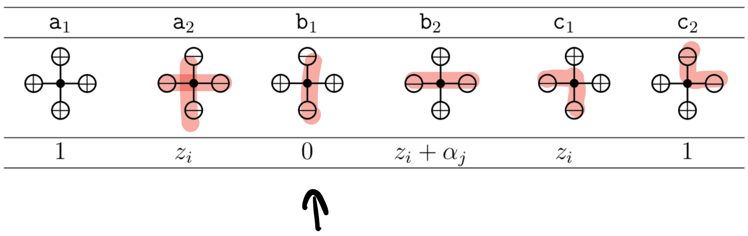
SOMETIMES TWO DIFFERENT SYSTEMS CAN HAVE THE SAME PARTITION FUNCTION.

REF: NOTES CH. 4 SECTION 7.

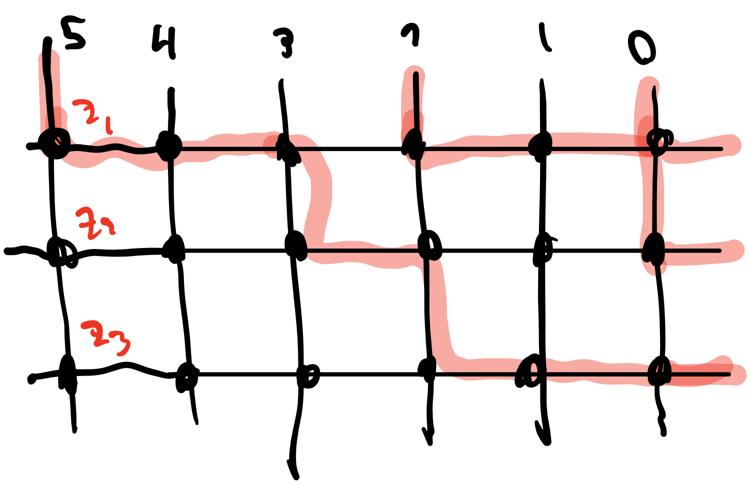
A PRECISELY ANALOGOUS RELATIONSHIP IS FOUND BETWEEN CLASSICAL AND BUMPLESS PIPEDREAMS.

Γ



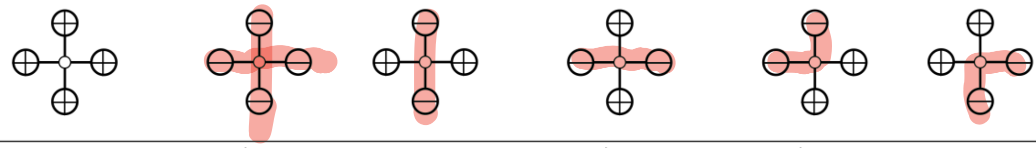
$q=0$ TOKUYAMA MODELS WITH α_j PARAM INTRODUCED.

PUT ⊖ IN COLUMNS $\lambda_i + n - i$ AND RIGHT EDGE



$\lambda = (3,1,0)$

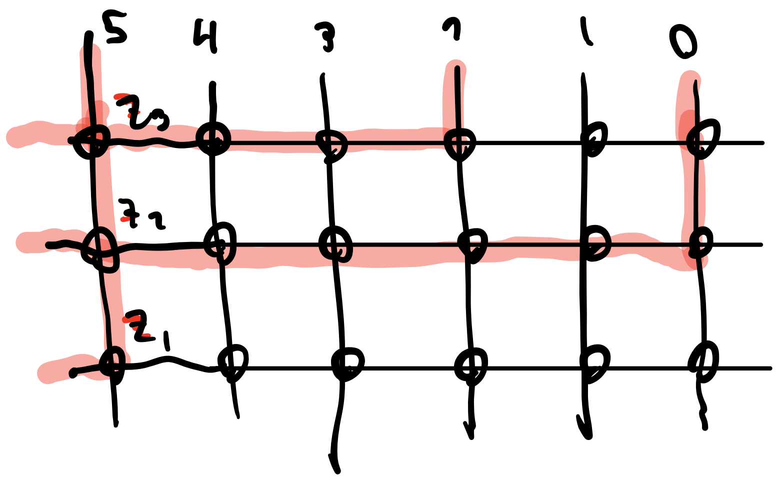
$\lambda + \rho = (5,2,0)$



a_1	a_2	b_1	b_2	d_1	d_2
$z_i + \alpha_j$	1	$-\alpha_j$	1	1	z_i

PATHS MOVE DOWN AND LEFT.
PUT ⊖ IN COLUMNS $\lambda_i + n - i$
AND ~~RIGHT~~ EDGE
LEFT

$n = 3$.



λ_i ARE ON $n-i$ ROW.

THEOREM: $Z(\mathfrak{S}^{\Gamma}_{\lambda}(z)) = Z(\mathfrak{S}^{\Delta}_{\lambda}(z))$.

ONE APPROACH WOULD BE TO EVALUATE THEM

$$Z = z^{\rho} \cdot \Delta_{\lambda}(z;\alpha)$$

"FACTORIAL SCHUR FUNCTIONS".

TODAY:

A DIRECT PROOF BASED ON YANG BAXTER EQUATIONS

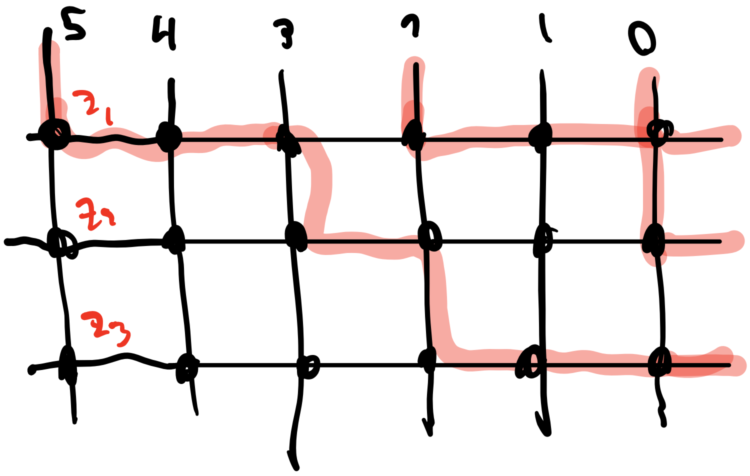
THIS LEADS TO HYBRID MODELS IN WHICH LAYERS OF Γ AND Δ ICE ARE MIXED TOGETHER.

a_1	a_2	b_1	b_2	c_1	c_2
⊕ / ⊕ ⊕ / ⊕	⊖ / ⊖ ⊖ / ⊖	⊖ / ⊕ ⊕ / ⊖	⊕ / ⊖ ⊖ / ⊕	⊕ / ⊖ ⊕ / ⊖	⊖ / ⊕ ⊖ / ⊕
1	z_i	0	$z_i + \alpha_j$	z_i	1

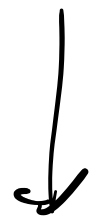
The Boltzmann weights for the *Delta model* are given as follows.

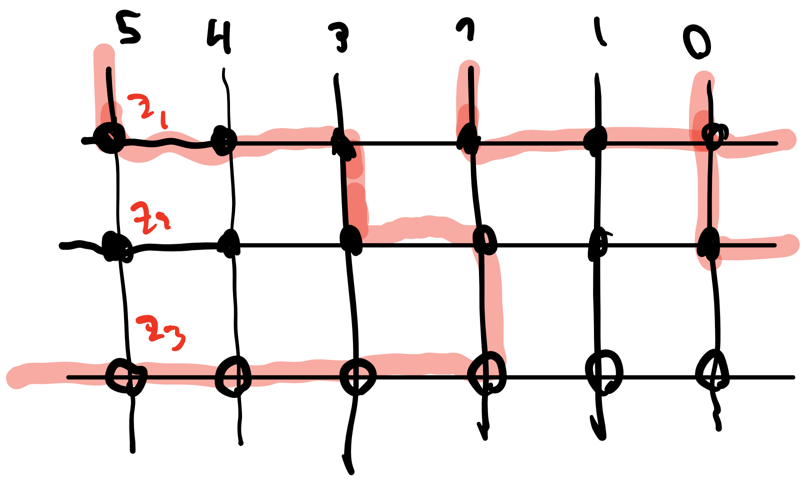
a_1	a_2	b_1	b_2	d_1	d_2
⊕ / ⊕ ⊕ / ⊕	⊖ / ⊖ ⊖ / ⊖	⊖ / ⊕ ⊕ / ⊖	⊕ / ⊖ ⊖ / ⊕	⊖ / ⊖ ⊕ / ⊕	⊕ / ⊕ ⊖ / ⊖
$z_i + \alpha_j$	1	$-\alpha_j$	1	1	z_i

OBSERVATION: WE MAY TRANSFORM THE BOTTOM ROW FROM Γ TYPE TO Δ



TAKE A STATE





SWITCH ⊕ TO ⊖ IN BOTTOM ROW.

Γ AND Δ WEIGHTS ARE DIFFERENT BUT WE ARE ONLY CONCERNED WITH A SITUATION WHERE ⊕ IS BELOW THE VERTEX.

a_1	a_2	b_1	b_2	c_1	c_2
1	z_i	0	$z_i + \alpha_j$	z_i	1

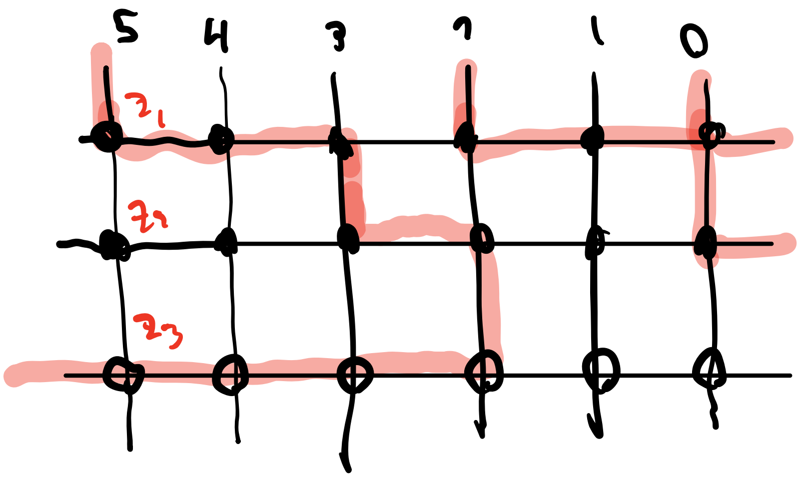
The Boltzmann weights for the *Delta model* are given as follows.

a_1	a_2	b_1	b_2	d_1	d_2
$z_i + \alpha_j$	1	$-\alpha_j$	1	1	z_i

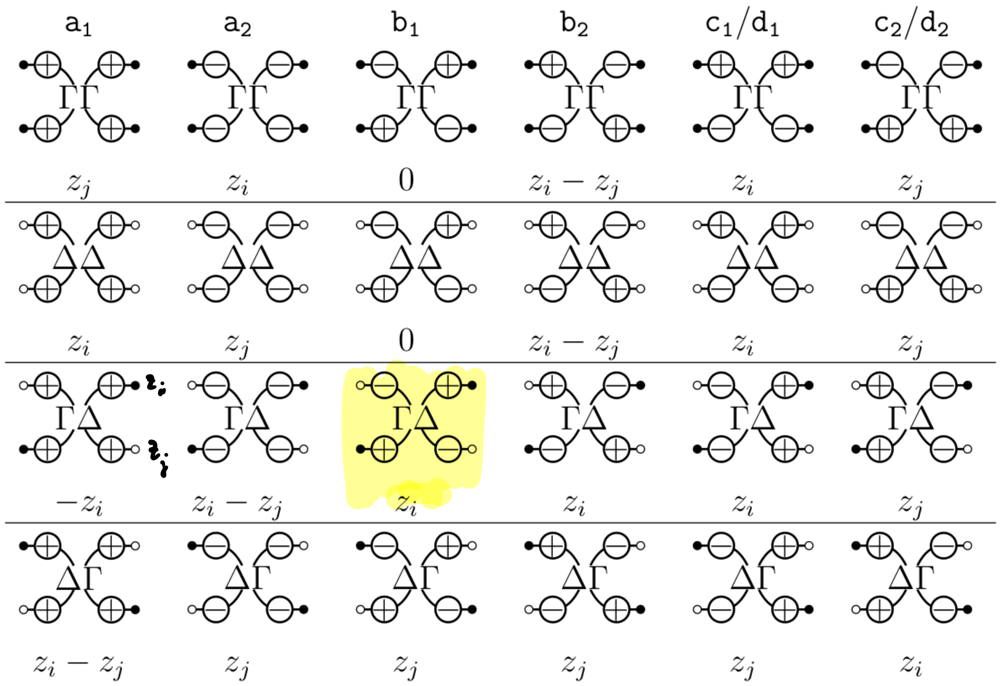
ONLY a_1, b_1, c_2 CAN OCCUR IN BOTTOM ROW

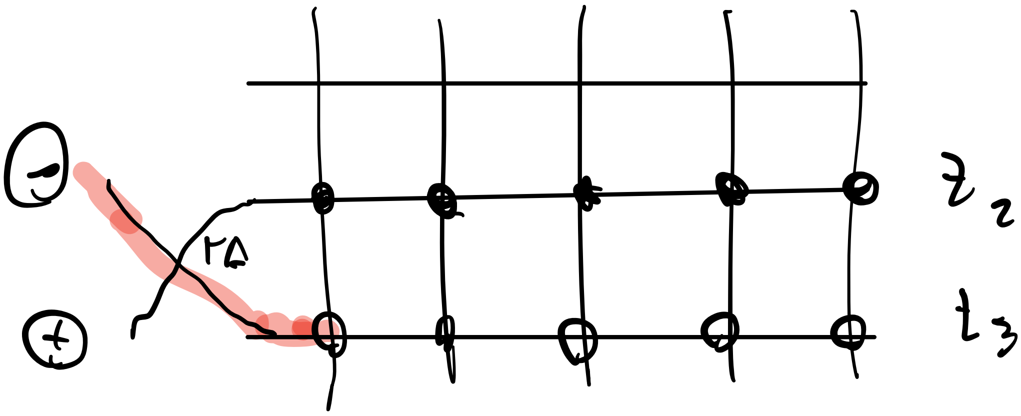
REVERSING SPINS ON HORIZONTAL EDGES TURN THESE INTO b_2, a_1, d_1 WITHOUT CHANGING THE WEIGHTS.

WE CAN CHANGE EVERY STATE OF $\mathfrak{S}^{\Gamma}$ INTO A STATE OF A HYBRID MODEL.

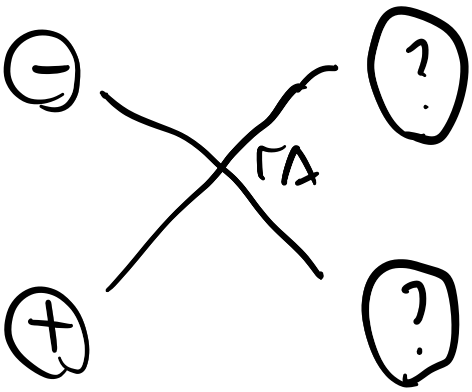


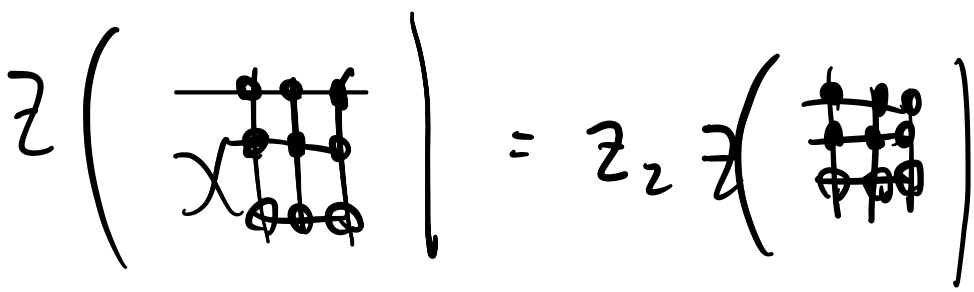
USE THIS R-MAT.

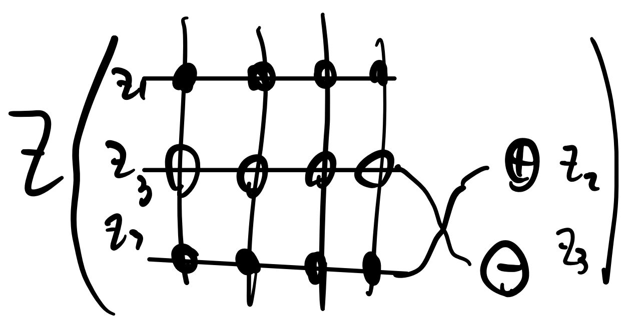




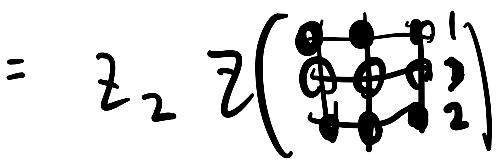
OBSERVE GIVEN SPINS



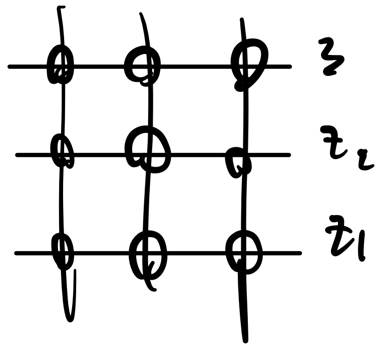




TRAIN

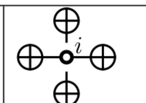
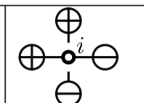
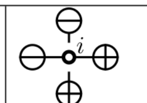
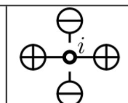
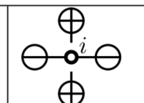
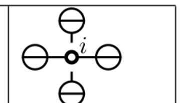
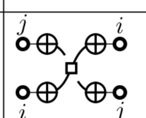
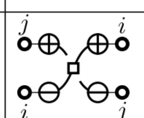
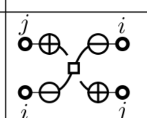
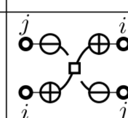
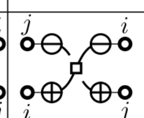
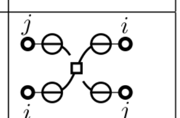
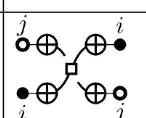
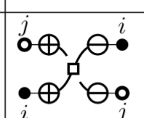
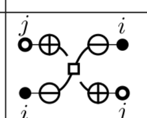
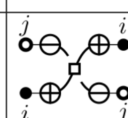
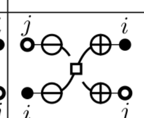
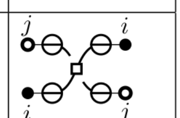
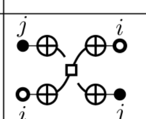
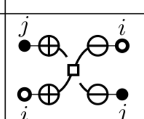
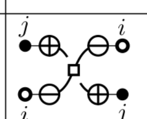
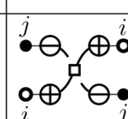
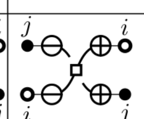
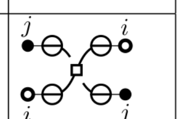


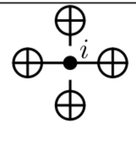
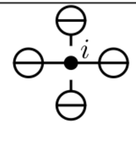
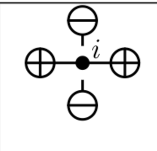
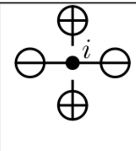
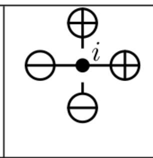
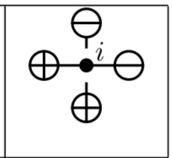
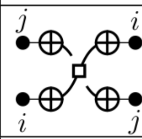
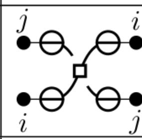
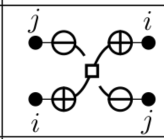
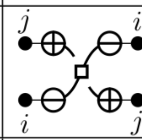
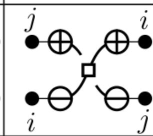
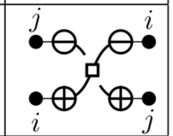
IF WE REPEAT THE PROCESS WE CAN MOVE Δ LAYER TO THE TOP, GO BACK AND CHANGE ANOTHER Γ LAYER, REPEAT.

$= Z\Big($  $\Big)$

AFTER $\frac{1}{2}n(n-1)$ SUCH OPERATIONS.

IN SCHUR POLYNOMIALS AND THE YANG-BAXTER EQUATION (BRUBAKER, BUMP, FRIEDBERG)

Delta Ice						
Boltzmann weight	z_i	$z_i(t_i+1)$	1	$z_i t_i$	1	1
Delta-Delta R-ice						
Boltzmann weight	$t_i z_i + z_j$	$z_j(t_j+1)$	$t_j z_j - t_i z_i$	$z_i - z_j$	$(t_i+1)z_i$	$z_i + t_j z_j$
Gamma-Delta R-ice						
Boltzmann weight	$t_i t_j z_j - z_i$	$(t_j+1)z_j$	$t_i z_j + z_i$	$t_j z_j + z_i$	$(t_i+1)z_i$	$z_i - z_j$
Delta-Gamma R-ice						
Boltzmann weight	$z_i - z_j$	$(t_i+1)z_i$	$t_j z_i + z_j$	$t_i z_i + z_j$	$(t_j+1)z_j$	$-t_i t_j z_i + z_j$

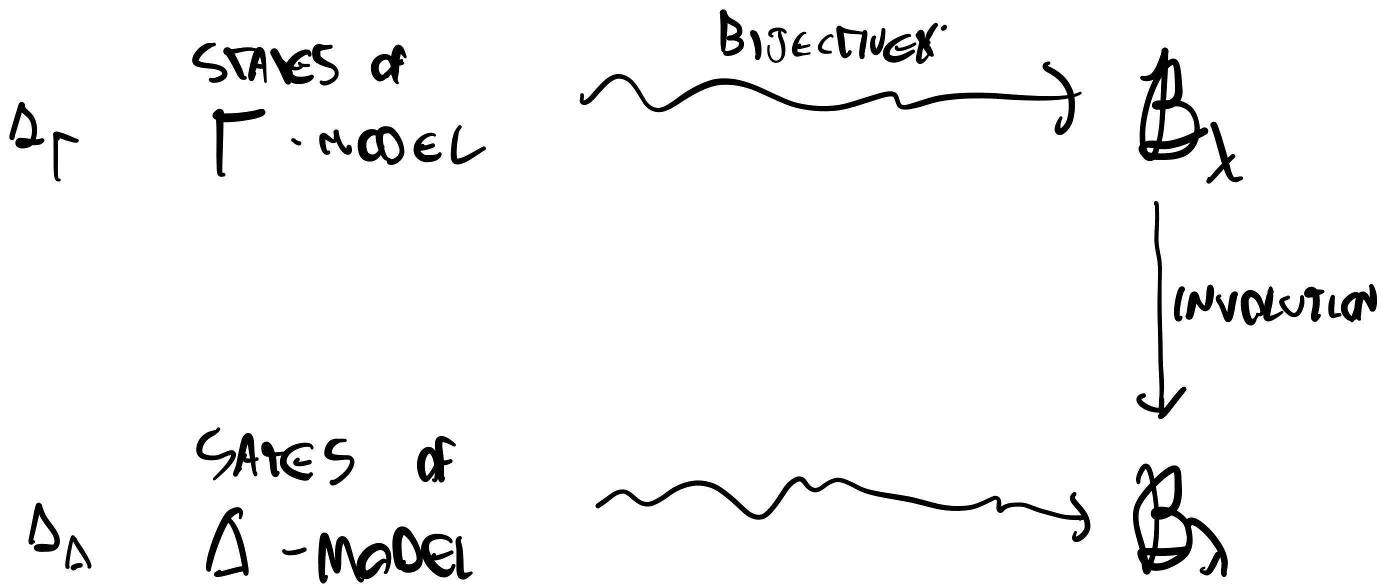
Gamma Ice						
Boltzmann weight	1	z_i	t_i	z_i	$z_i(t_i+1)$	1
Gamma-Gamma R-ice						
Boltzmann weight	$t_j z_i + z_j$	$t_i z_j + z_i$	$t_i z_j - t_j z_i$	$z_i - z_j$	$(t_i+1)z_i$	$(t_j+1)z_j$

THIS SAME ARGUMENT IS GIVEN WITH MORE GENERAL WEIGHTS ($t_i = -q$)

IN BBF WEYL GROUP MDS (ANNALS OF MATHEMATICS STUDIES) EVEN MORE GENERAL PARTITION FUNCTIONS ARE CONSIDER.

$q = 0$ MODEL CAN BE WITHOUT α_i

STATEMENT CAN BE RELATED TO CRYSTAL.



$$B(\Delta_\Gamma) = B(\Delta_\Delta).$$

FOR THIS CASE THE INVOLUTION CAN BE USED BUT NOT IN MORE GENERAL CASES.