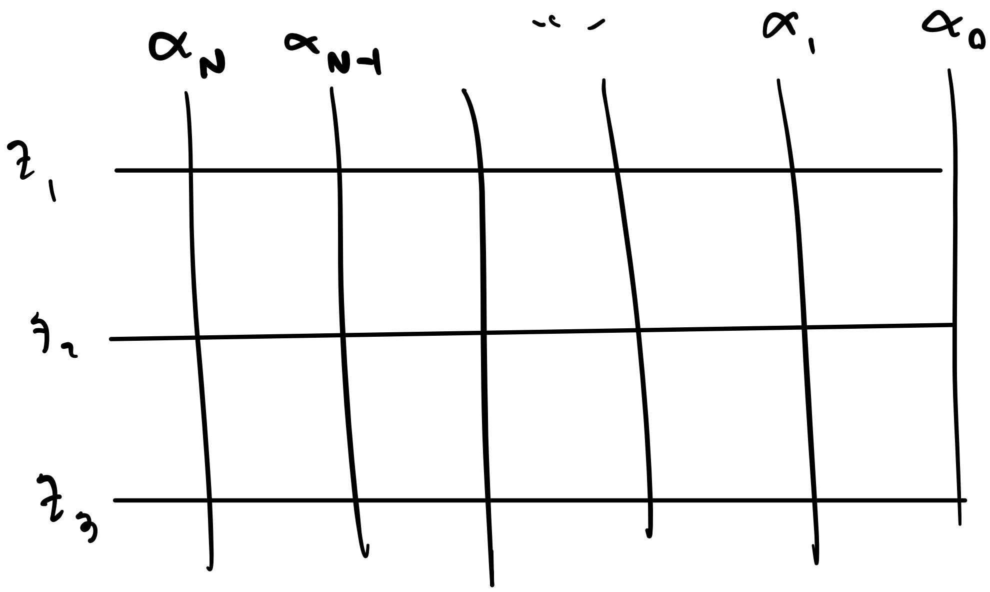
COLUMN PARAMETERS

HYBRID MODELS.



IN MANY CASES COLUMN PARAMETERS CAN BE INTRODUCED FOR FREE LEADING TO (FOR EXAMPLE)

FACTORIAL SCHUR FUNCTIONS

DOUBLE SCHUBERT POLYNOMIALS.

CONSIDER WHEN THE BOLTZMANN WEIGHTS COME FROM A PARAMETRIZED YANG-BAXTER EQUATION.

Γ = A GROUP, V A VECTOR SPACE

$$R : \Gamma \longrightarrow \mathrm{END}(V \otimes V)$$

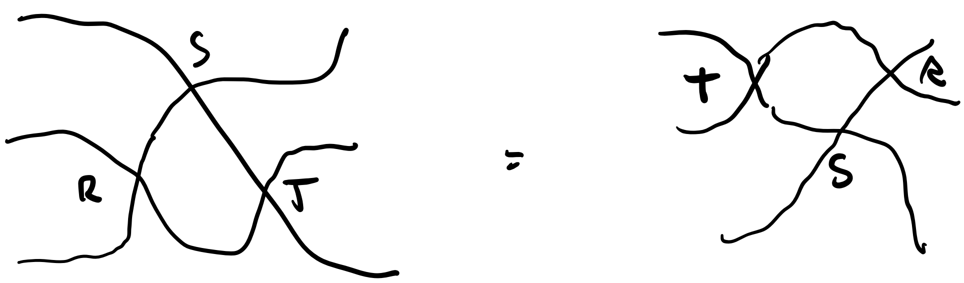
SUCH THAT IN $\mathrm{END}(V \otimes V \otimes V)$

$$[[R(\gamma), R(\gamma\delta), R(\delta)]] = 0$$

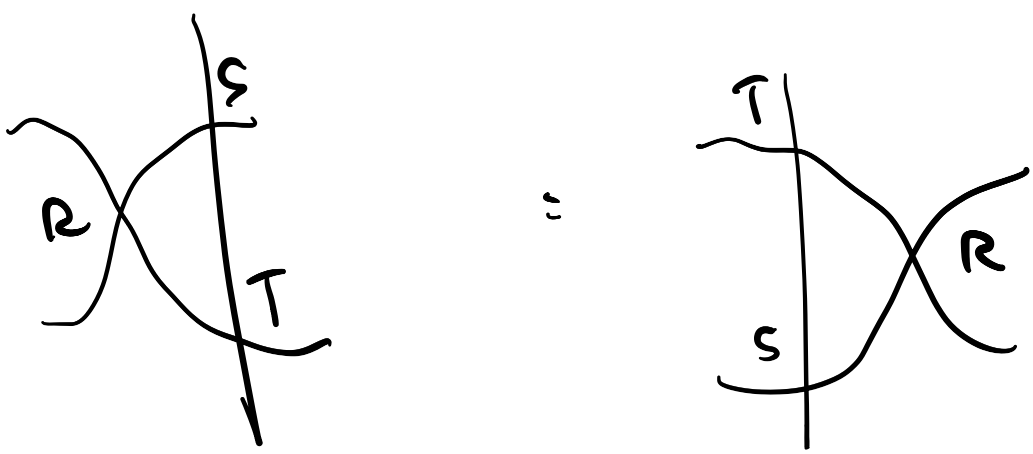
$$[[R, S, T]] =$$

$$(R \otimes I)(I \otimes S)(T \otimes I) - (I \otimes T)(S \otimes I)(I \otimes R)$$

EQUIVALENT TO A YBE:

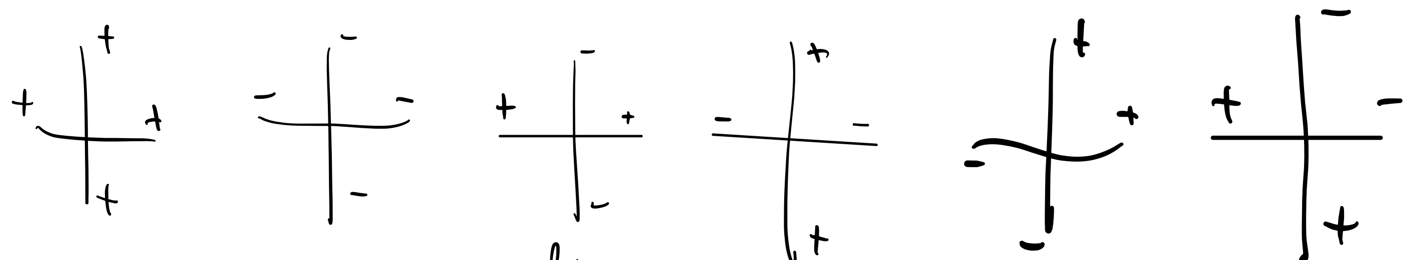


$$[[R, S, T]] = 0$$



EXAMPLE 1: (BAXTER)

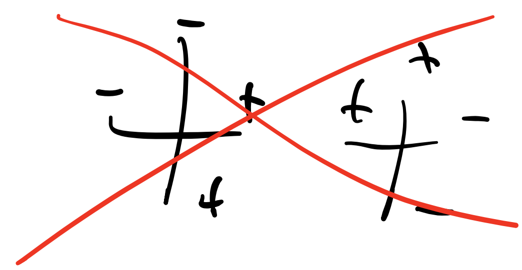
LET US FIX Δ AND CONSIDER BOLTZMAN WEIGHTS:



a_1 a_2 b_1 b_2 c_1 c_2

"FIELD-FREE"

$$a_1 = a_2 = a$$
$$b_1 = b_2 = b$$
$$c_1 = c_2 = c$$



$$\Delta(\theta) = \frac{a^2 + b^2 - c^2}{2ab}$$

THEOREM (BAXTER) IF S, T ARE GIVEN $\Delta(S) = \Delta(T) = \Delta$ THERE EXISTS R WITH

$$[[R, S, T]] = 0 \quad \text{AND} \quad \Delta(R) = \Delta.$$

WE CAN THEN FIND A MAP

$$R : \mathbb{C}^\times \to \mathrm{END}(V \otimes V)$$

V = 2-DIM'L WITH BASIS v_+, v_-

SUCH THAT

$$S = R(\gamma\delta), \quad T = R(\delta)$$

FOR $\gamma, \delta \in \mathbb{C}^\times$.

ADDED AFTER THE LECTURE. SEE NOTE THM 3.10,

$$(a, b, c) = \left(\tfrac{1}{2}(xq - (xq)^{-1}), \tfrac{1}{2}(x - x^{-1}), \tfrac{1}{2}(q - q^{-1})\right).$$

USE THESE WEIGHTS FOR $R(x)$

SIX-VM NOT ASSUMING FIELD FREE

EXAMPLE 2: THE WEIGHTS ARE CALLED FREE-FERMIONIC IF

$$a_1 a_2 + b_1 b_2 = c_1 c_2 .$$

THEOREM: IF S, T ARE FREE-FERMIONIC THERE IS ALWAYS AN R SUCH THAT

$$[[R, S, T]] = 0$$

FELDERHOF.
(KOREPIN, BRUBAKER-BUMP-FRIEDBERG)

MOREOVER THERE IS A PYBE

$$\Gamma = GL(2,\mathbb{C}) \times \mathbb{C}^{*}$$

WHOSE IMAGE IS EXACTLY THE FREE FERMIONIC CASE.

QUANTUM GROUPS:

EXAMPLE 1: $U_q(\mathfrak{sl}_2)$

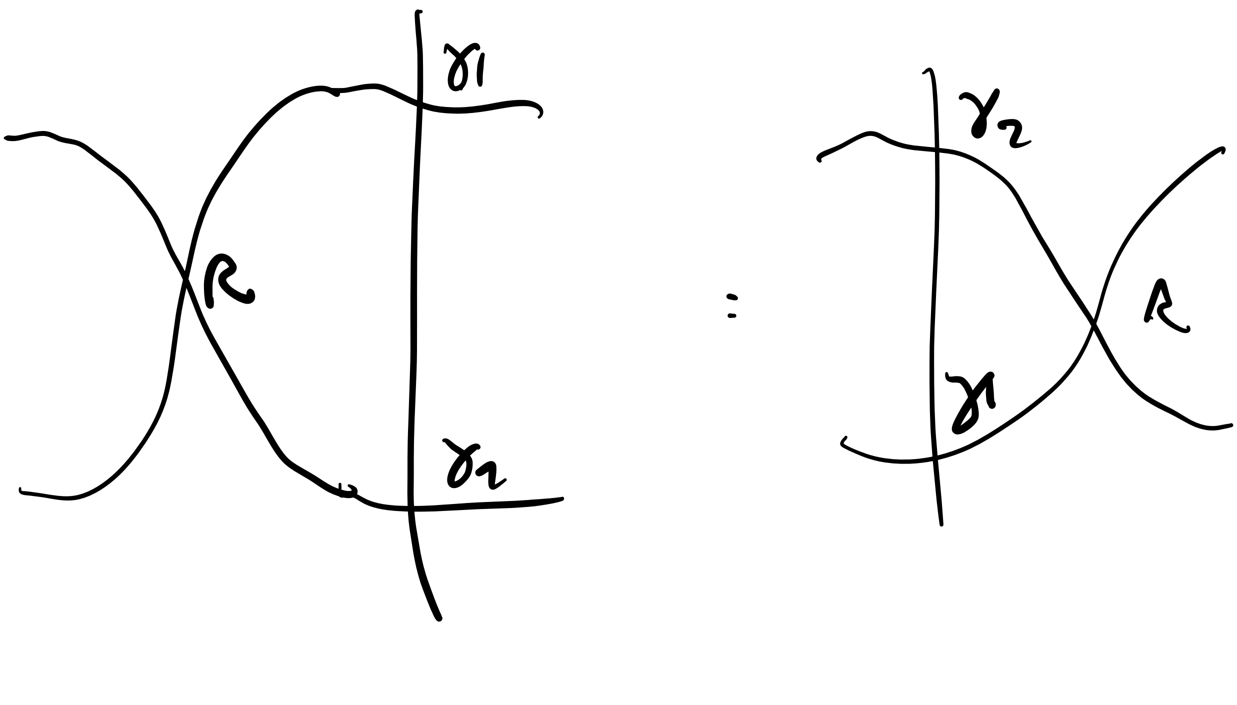
EXAMPLE 2: $U_q(\mathfrak{sl}(1|1))$.

FREE-FERMIONIC

TOKUYAMA WEIGHTS ARE FREE-FERMIONIC.

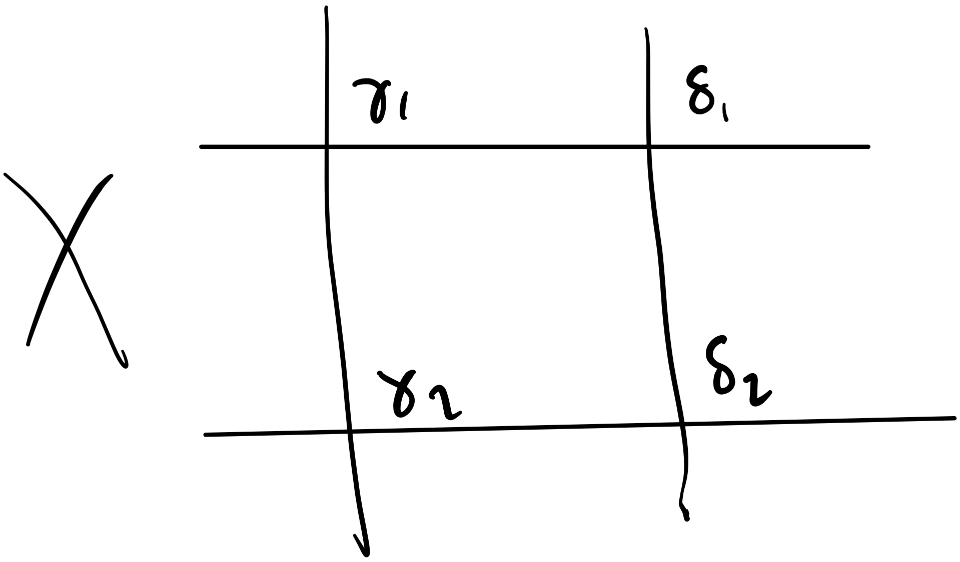
WE CAN CHOOSE $z_1, \dots, z_n \in \Gamma$

AND ASK FOR AN R-MATRIX
THAT WILL MAKE TRAIN ARGUMENT

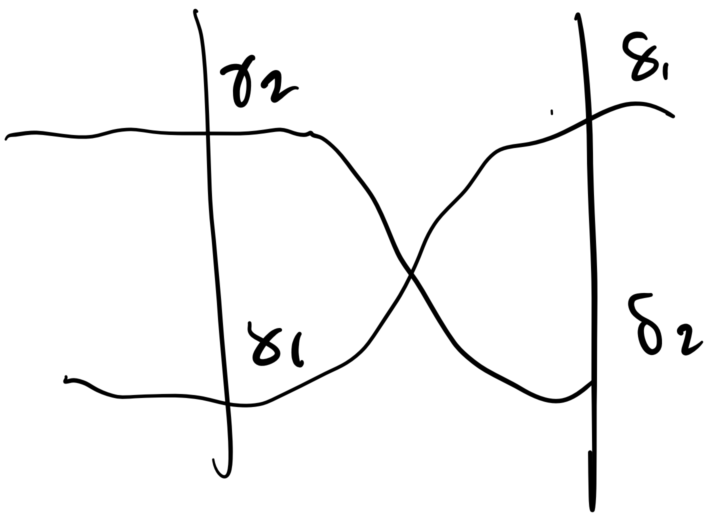


$\gamma_1 = R \cdot \gamma_2$ (IN Γ, POSSIBLY NON ABELIAN

$R = \gamma_2 \gamma_1^{-1}$



TO REPEAT THE OPERATION



$= \ldots$

I NEED $\gamma_1 \gamma_2^{-1} = \delta_1 \delta_2^{-1}$.

WHAT I CAN DO IS CHOOSE

$$z_1, \dots, z_n \in \Gamma$$

$$\alpha_1, \dots, \alpha_n \in \Gamma$$

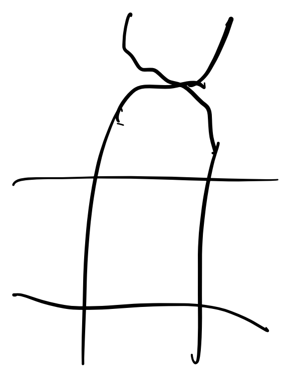
$$\gamma_i = z_i \alpha_j$$

$$\begin{array}{c|c}
z_1\alpha_1 \ \text{"}\gamma_1\text{"} & z_1\alpha_2 \ \text{"}\delta_1\text{"} \\
\hline
z_2\alpha_1 \ \text{"}\gamma_2\text{"} & z_2\alpha_2 \ \text{"}\delta_2\text{"} \\
\hline
\end{array}$$

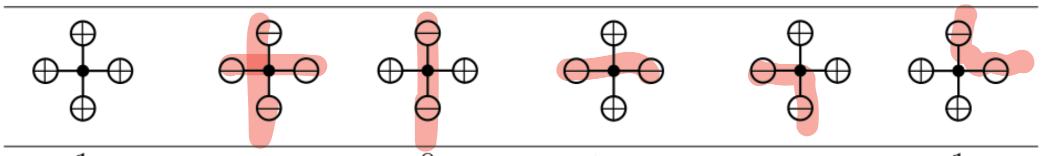
$$\gamma_2\gamma_1^{-1} = z_2\cancel{\alpha_1\alpha_1^{-1}}z_1^{-1} = \delta_2\delta_1^{-1}$$

I CAN REPEAT THE TRAIN ARGUMENT.

BONUS: WITH THIS CHOICE WE ALSO GET VERTICAL TBE



a_1	a_2	b_1	b_2	c_1	c_2
1	z_i	0	$z_i + \alpha_j$	z_i	1



BY THIS METHOD WE CAN INTRODUCE COLUMN PARAMETERS INTO THE $q=0$ TOKUYAMA MODELS. (EVEN $q \neq 0$.)

$$Z(\mathfrak{S}_\lambda(z;\alpha)) = z^{\rho}\Delta_\lambda(z;\alpha)$$

WHERE $\Delta_\lambda(z;\alpha)$ IS SYMMETRIC IN z,

NOT QUITE SYMMETRIC IN α.

"ASYMPTOTICALLY SYMMETRIC"

IF I FIX A PERMUTATION OF α:

$$\Delta_\lambda(z;w\alpha) = \Delta_\lambda(z;\alpha)$$

PROVIDED $n = \#$ ROWS IS LARGE DEPENDING ON w.

I EXPLAINED IN LECTURE 1

SCHUR POLYNOMIALS ARE RELATED TO CLASSES OF SCHUBERT VARIETIES IN

$$H^*(\text{GRASSMANNIAN}) \cong \text{SYMMETRIC POLY} / \text{IDEAL}$$

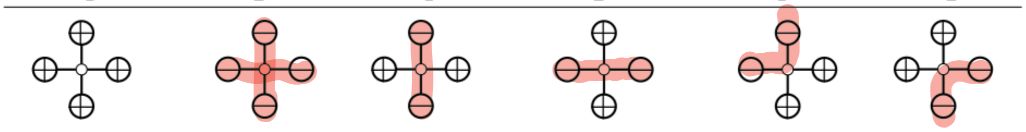
THE FACTORIAL SCHUR POLYNOMIALS CORRESPOND TO EQUIVARIANT COHOMOLOGY.

$$\lambda = (\lambda_1, \dots, \lambda_n, 0, \dots, 0)$$

$$\Delta_\lambda(z_1, \dots, z_n, 1, \dots, 1) =$$

$$\Delta_\lambda(z_1, \dots, z_n)$$

Δ



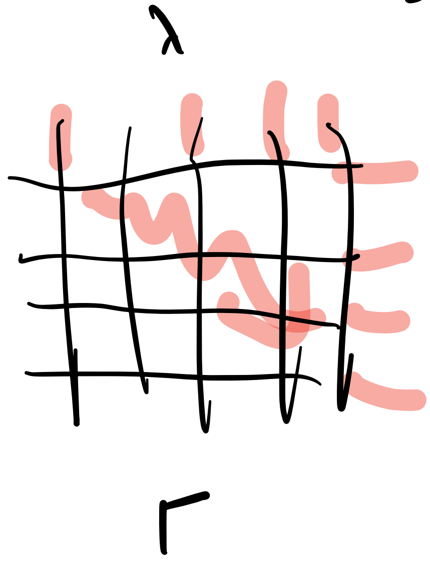
a_1	a_2	b_1	b_2	d_1	d_2
$z_i + \alpha_j$	1	$-\alpha_j$	1	1	z_i

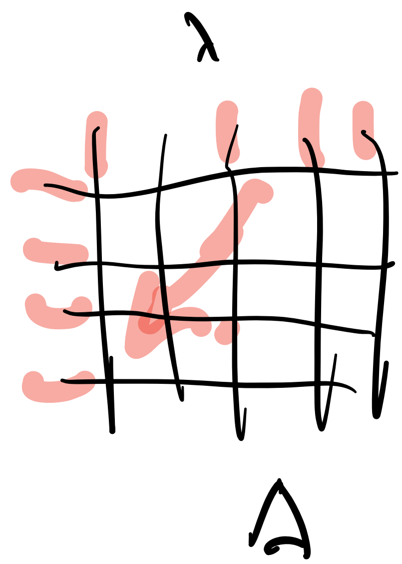
THERE IS A SECOND TYPE OF MODEL

$S^{\Gamma}_{\lambda}(z; \alpha)$: $q = 0$ TOKUYAMA MODEL.

$S^{\Delta}_{\lambda}(z; \alpha)$ IS SIMILAR

BUT PATHS ~~MOVE~~ DOWN AND TO THE LEFT. SO WE MODIFY THE BOUNDARY CONDITIONS:





IN BOTH CASES

PUT - SPINS AT SAME $\lambda_i + n - i$ LOCATIONS ON TOP.

THEOREM:

$$Z\left(S_\lambda^{\Gamma}(z;\alpha)\right) = Z\left(S_\lambda^{\Delta}(z;\alpha)\right).$$

WE CAN USE YBE TO PROVE THIS. THESE IDEAS WILL RESURFACE WHEN WE DISCUSS KNUTSON-UDELL HYBRID MODELS FOR PIPEDREAMS.