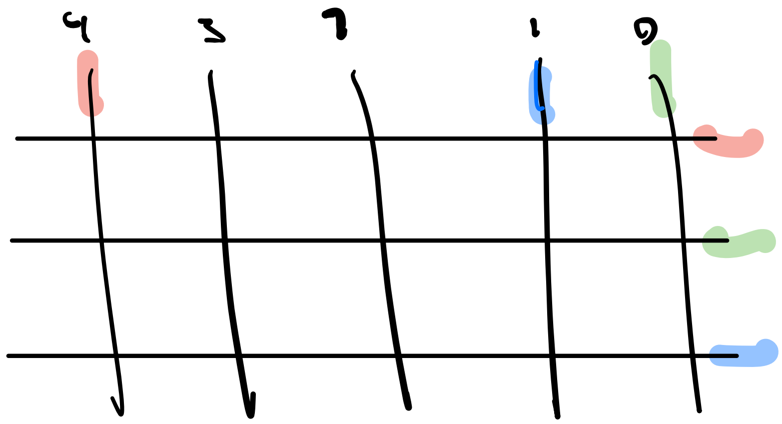
OPEN AND CLOSED MODELS CONTINUED

LAST TIME WE SAW HOW YANG-BAXTER EQUATION CAN LEAD TO RECURSIONS INVOLVING DEMAZURE OPERATORS



$\lambda = (3, 1, 0)$

$R > B > G$

λ PUT COLOR C_{n-i} AT $\lambda_i + n - i$

(DECREASING FROM LEFT TO RIGHT

RIGHT BOUNDARY COLORS ARE VARIABLE AND DETERMINE A PERMUTATION w

$C_{w^{-1}(n-i)}$ IN i-TH ROW

$w = (23)$ IN EXAMPLE.

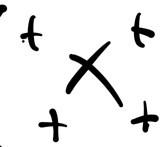
$$a(z_i, z_{i+1})\, Z(\mathfrak{S}_{\lambda,w}(z)) =$$

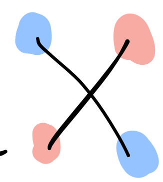
$$b(z_i, z_{i+1})\, Z(\mathfrak{S}_{\lambda, s_i w}(s_i z)) +$$

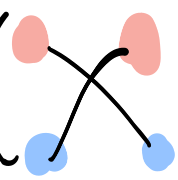
$$c(z_i, z_{i+1})\, Z(\mathfrak{S}_{\lambda,w}(s_i z)).$$

REPLACE $z \to s_i z$ $\qquad z_i \leftrightarrow z_{i+1}$

$$Z(\mathfrak{S}_{\lambda, s_i w}(z)) = \frac{c(z_{i+1}, z_i)\, Z(\mathfrak{S}_{\lambda, w}(z)) - a(z_{i+1}, z_i)\, Z(\mathfrak{S}_{\lambda, w}(s_i z))}{-b(z_{i+1}, z_i)}$$

$a(z,w) = \beta\Big($  $\Big)$

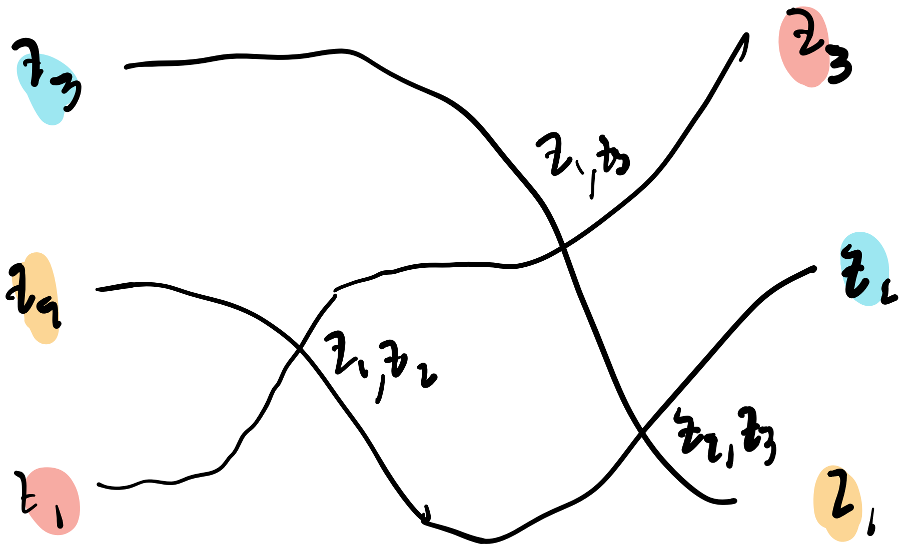
$b(z,w) = \beta\Big($  $\Big) = \text{const} \times (z - w)$

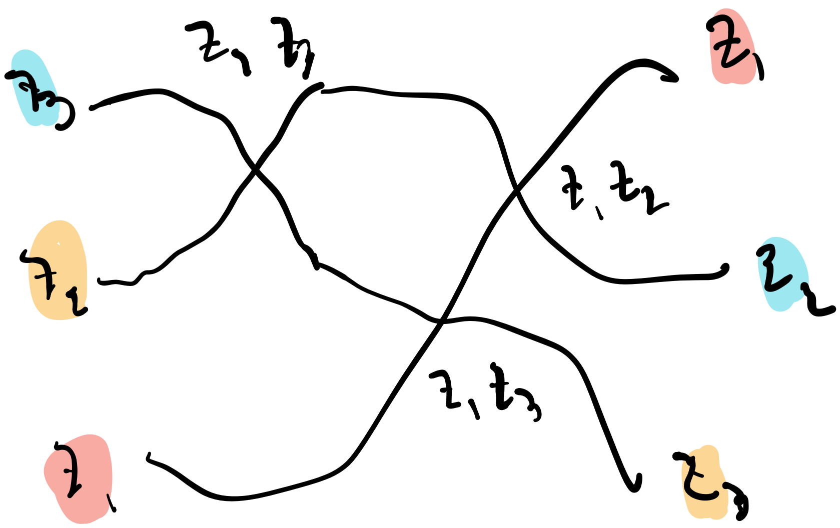
$c(z,w) = \beta\Big($  $\Big)$

THE RMATRIX IS AN ENDOMORPHISM OF $V \otimes V$

V = FREE-VECTOR SPACE ON THE SET OF SPINS

$$
\begin{tikzcd}
 & V \otimes V \otimes V \arrow[dl, "R(z_1, z_2) \otimes I"'] \arrow[dr, "I \otimes R(z_2, z_3)"] & \\
V \otimes V \otimes V \arrow[d, "I \otimes R(z_1, z_3)"'] & & V \otimes V \otimes V \arrow[d, "R(z_1, z_3) \otimes I"] \\
V \otimes V \otimes V \arrow[dr, "R(z_2, z_3) \otimes I"'] & & V \otimes V \otimes V \arrow[dl, "I \otimes R(z_1, z_2)"] \\
 & V \otimes V \otimes V &
\end{tikzcd}
$$





THE EQUIVALENCE OF THESE TWO INTERPRETATIONS OF Y.B.E. ARE IN SECTION 1.6 OF NOTES.

ON FRIDAY WE CONSIDERED OPEN MODELS. THE ABOVE FORMALISM GIVES

IF $\Delta_i w > w$

$$Z(S_{\lambda, \Delta_i w}) = {}^{p}\partial_i^{0} Z(S_{\lambda, w})$$

REMEMBER $\partial_i^{0} = (z^{\alpha_i} - 1)^{-1}(1 - \Delta_i)$

SATISFY BRAID RELATION SO

$$\partial_w^{0} = \partial_{i_1} \cdots \partial_{i_k}$$

$$\Delta = \Delta_{i_1} \cdots \Delta_{i_r}$$

$$z^{p} D z^{-p} = {}^{p}D \qquad z^{p} \Delta_i z^{-p} = z^{\alpha_i}$$

$${}^{p}\partial_i^{0} = (z^{\alpha_i} - 1)^{-1}(1 - z^{\alpha_i}\Delta_i)$$

$$Z(\mathfrak{S}_{\lambda,1}) = z^{\lambda+\rho}$$

BY INDUCTION

$$Z(\mathfrak{S}_{\lambda,w}) = z^{\rho}\, \underbrace{\partial^{\circ}_{w} z^{\lambda}}_{\text{DEMAZURE ATOM}}$$

$$\Delta_\lambda(z) = \sum_{w \in W} \partial^{\circ}_{w} z^{\lambda}$$

THIS HAS AN ANALOG FOR THE CRYSTAL

$\mathcal{B}_\lambda$ = CRYSTAL OF SSYT OF SHAPE λ

$$\sum_{w \in W} z^{wt(T)} = \Delta_\lambda(z)$$

THIS IS A SUM OVER STATES OF THE $q=0$ TOKUYAMA MODEL.

THM (BRUBAKER, BUCIUMAS, BUMP, GUSTAFSSON)

THE STATES OF THE OPEN MODEL ARE IN BIJECTION WITH A SUBSET

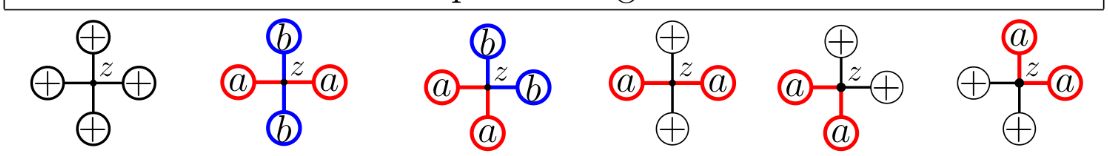
$$\mathcal{B}^{\circ}_{\lambda}(w)$$

CALLED A DEMAZURE ATOM.

THE ARGUMENTS USING YBE DON'T QUITE PROVE BUT THEY DO SHOW THIS PREDICTS RIGHT PARTITION FN

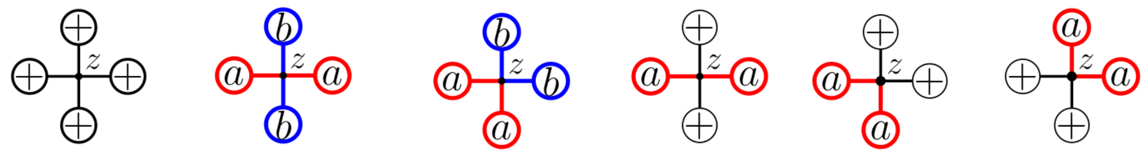
$$z^{\rho}\,\partial^{\circ}_{w}\,z^{\lambda}.$$

Open T-weights

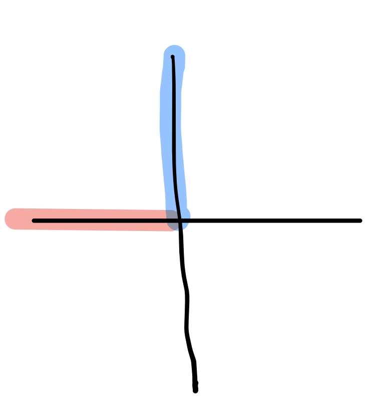


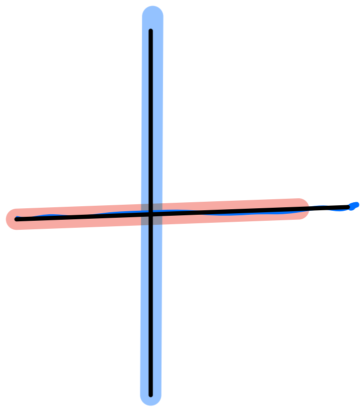
1	$z \quad a \geqslant b$ $0 \quad a < b$	$0 \quad a > b$ $z \quad a < b$	z	z	1

Closed T-weights



1	$z \quad a \geqslant b$ $0 \quad a < b$	$z \quad a > b$ $0 \quad a < b$	z	z	1

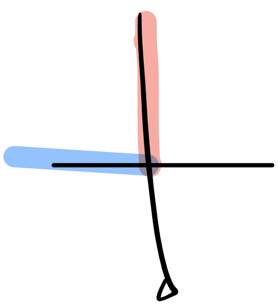


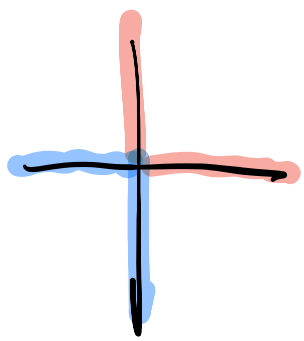


FORCED

OPEN

R > B

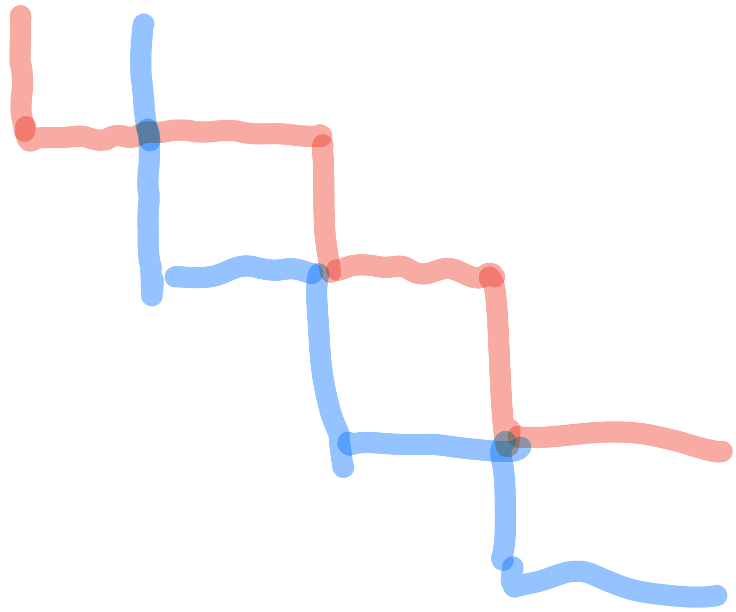




FORCED.

CONCLUSION: IF LARGER COLOR COMES IN FROM LEFT, THEY MUST CROSS.
IF SMALLER COLOR COMES IN FROM LEFT MAY NOT CROSS.

WHAT HAPPENS IF TWO PATHS CROSS SEVERAL TIMES



OPEN MODEL STATE.

FOR CLOSED MODEL:

IF $\Delta_{i_N} \omega \geq \omega$ THEN

$$Z(S_{\lambda, \Delta_{i_N}\omega}) = {}^{\rho}\partial_i Z(S_{\lambda,\omega})$$

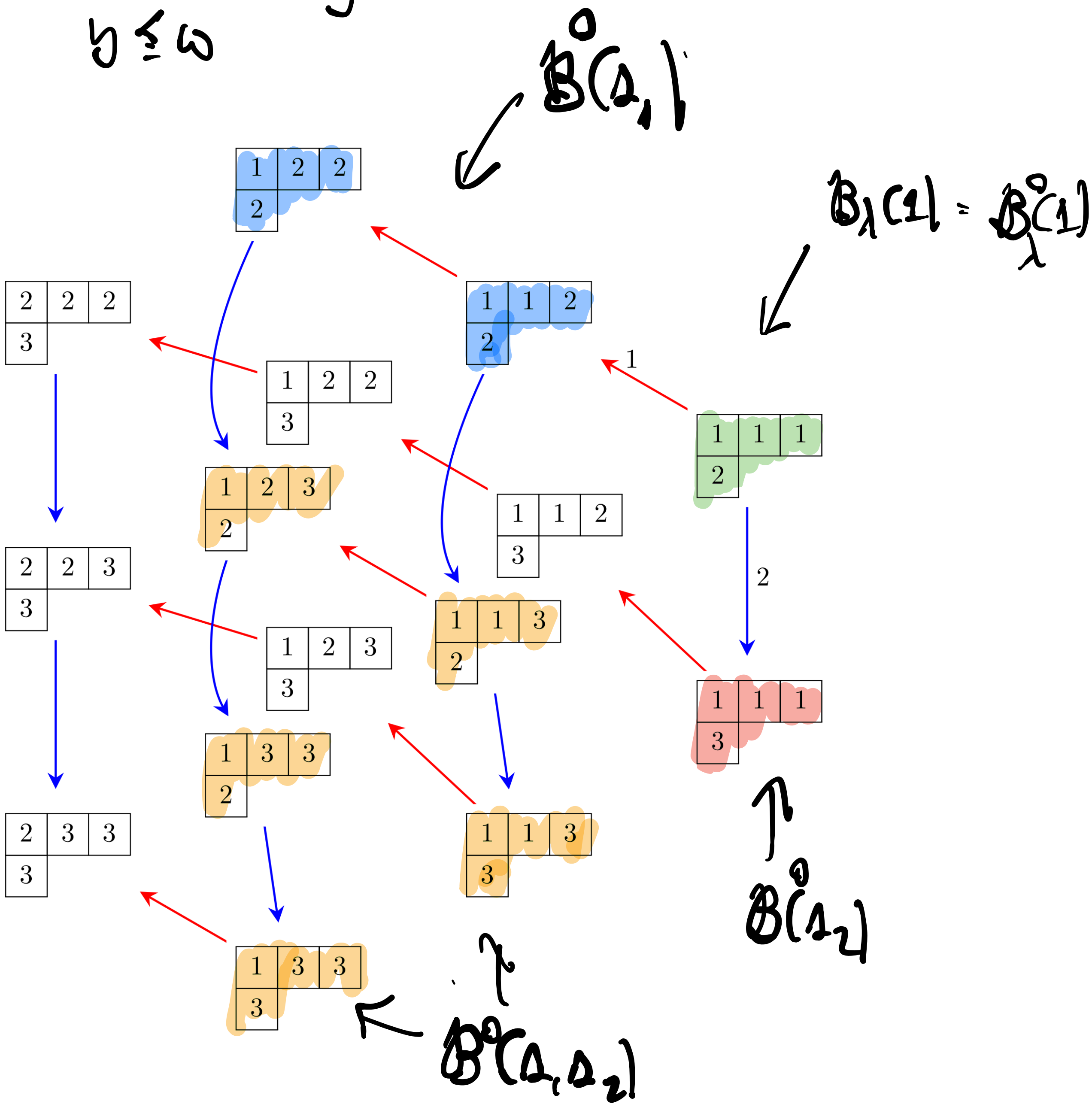
SO $$Z(S_{\lambda,\omega}) = z^{\rho} \underbrace{\partial_\omega z^{\lambda}}_{\text{DERIVATIVE CHARACTER.}}$$

$$\partial_w z^\lambda = \sum_{\mathcal{B}_\lambda(w)} z^{wt(T)}$$

$\uparrow$

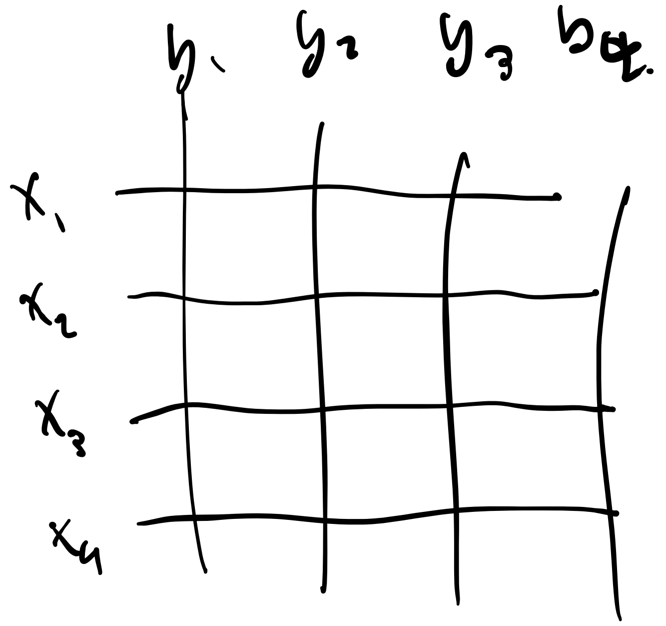
DEMAZURE CRYSTAL.

$$\partial_w = \sum_{y \leq w} \partial^\circ_y$$



THEOREM (YINGZI YANG) THE STATES OF THE CLOSED MODEL ARE BIJECTION WITH A DEMAZURE CRYSTAL.

PARAMETRIZED YANG-BAXTER EQUATIONS NATURALLY LEAD TO INTRODUCTION OF COLUMN PARAMETERS



PARAM-YBE: Γ A GROUP

$$R : \Gamma \to \mathrm{END}(V \otimes V)$$

$$[[R(z_1, z_2), R(z_1, z_3), R(z_2, z_3)]] = 0$$

IF $R, S, T \in \mathrm{END}(V \otimes V)$

$$[[R, S, T]] =$$

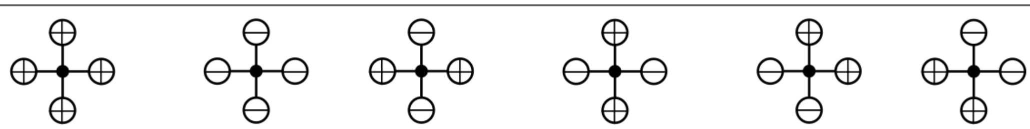
$$(R \otimes I)(I \otimes S)(T \otimes I) - (I \otimes T)(S \otimes I)(I \otimes R)$$

WE CAN CHOOSE $x_i \in \Gamma$, $y \in \Gamma$

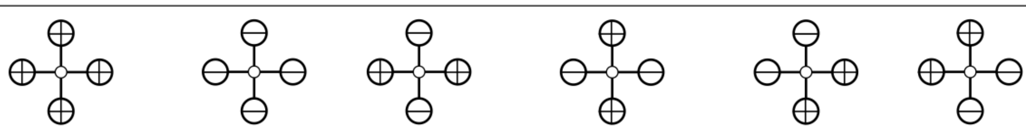
CHOOSE THE BOLTZMANN WEIGHTS

$$T_{i,j} = R(x_i, y_i).$$

a_1	a_2	b_1	b_2	c_1	c_2
1	z_i	0	$z_i + \alpha_j$	z_i	1



a_1	a_2	b_1	b_2	d_1	d_2
$z_i + \alpha_j$	1	$-\alpha_j$	1	1	z_i



HERE ARE TWO EXAMPLES z_i = ROW

α_j = COLUMN.

THIS WILL GIVE TWO SYSTEMS
WITH SAME PARTITION FUNCTION AND
INTERESTING STORY THAT PREFIGURES
A PHENOMENON IN PIPEDREAM SPACY.