

OPEN AND CLOSED MODELS (CONTINUED)

$$\lambda = (\lambda_1, \dots, \lambda_n) \text{ A PARTITION} \quad z = \begin{pmatrix} z_1 \\ \vdots \\ z_n \end{pmatrix}$$

$\Delta_\lambda(z)$ = PARTITION FUNCTION OF
 $q=0$ TAKUYAMA MODEL.

CLOURED VARIANTS (OPEN AND CLOSED)

REFLECT THE DIVISION OF THE CRYSTAL

\oplus_λ WHICH CHARACTER IS Δ_λ INTO
 DEMATURIE ATOMS (OPEN) OR
 DEMATURIE CHARACTERS (CLOSED).

THE YANG-BAXTER EQUATION CAN BE USED
 TO COMPUTE THE PARTITION FUNCTIONS.

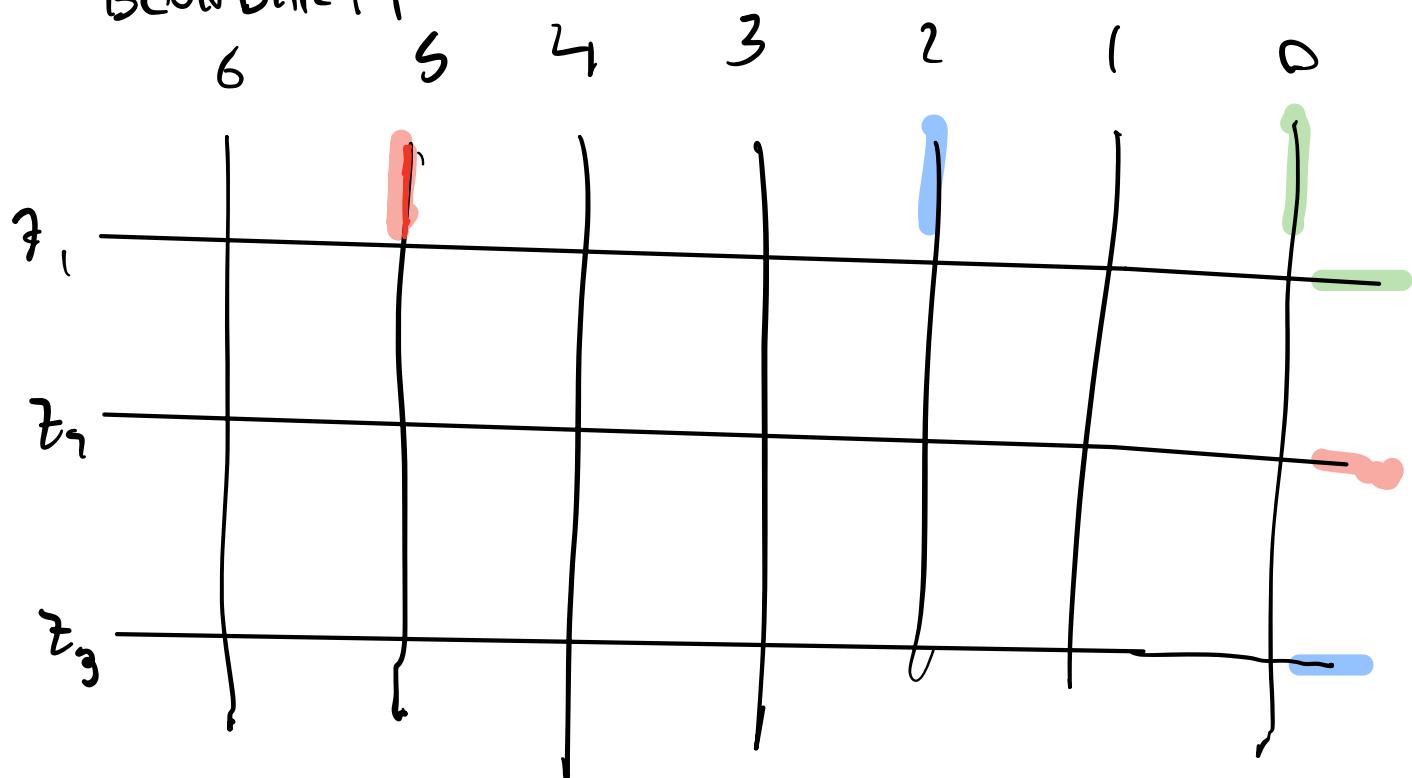
Open T-weights

1	$z \quad a \geq b$	0 $a > b$	z	z	1
0 $a < b$	$z \quad a < b$				

order colors $c_1 < c_2 < \dots < c_n$

But them in REVERSE order on top

BOUNDARY:



AT Col. $\lambda_i + n$ in PVT c_{n-i}

$$\lambda = (3, 1, 0) \quad \lambda + \rho = (5, 2, 0)$$

And for ~~RIGHT~~ BOUNDARY in row i

PUT $\omega \omega_0 c_0$ $\omega_0 c_0 = (c_n, c_{n-1}, \dots, c_1)$

$$\omega(d_1, \dots, d_n) = (d_{\omega^{-1}(1)}, \dots, d_{\omega^{-1}(n)})$$

$$\omega_1 > \Delta_1 \Delta_2 = (123)$$

$$c_3 > c_2 > c_1$$

THEOREM :

$$Z(S_{\lambda, \omega}) = Z^P \cdot \underbrace{\partial_{\omega}^0 Z^1}_{\text{DOM ATURE}} \cdot \text{ATOM}.$$

$$\partial_{\omega}^0 = \partial_{\omega_1}^0 \cdots \partial_{\omega_n}^0$$

$$\omega = \underbrace{\alpha_1 \cdots \alpha_n}_{\text{RECORDED}}$$

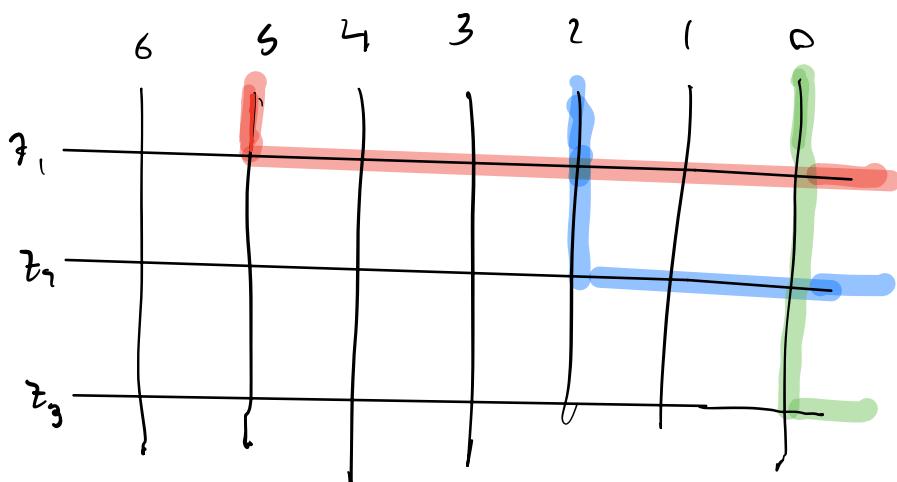
$$\partial_{\omega_i} = (Z^{\alpha_i} - 1)^{-1} (1 - \alpha_i)$$

BRAID.

PROVED BY INDUCTION ON $l(\omega)$.

IF $l(\omega) = 1$, THIS IS THE SYSTEM

$S_{\lambda, 1}$ WITH EXACTLY ONE STATE.



$$\begin{aligned} Z_1^5 Z_1^2 \\ = \prod Z_i^{\lambda_i + n - i} \\ = Z^{\lambda + P} \end{aligned}$$

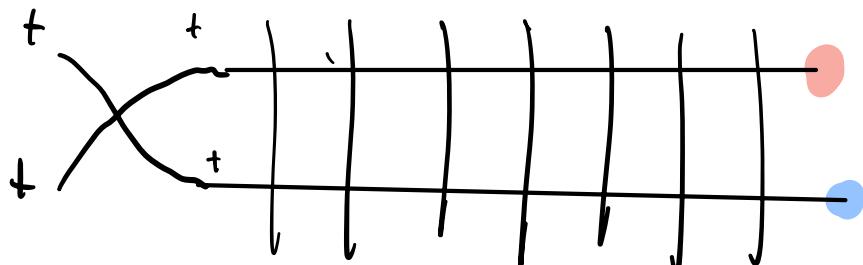
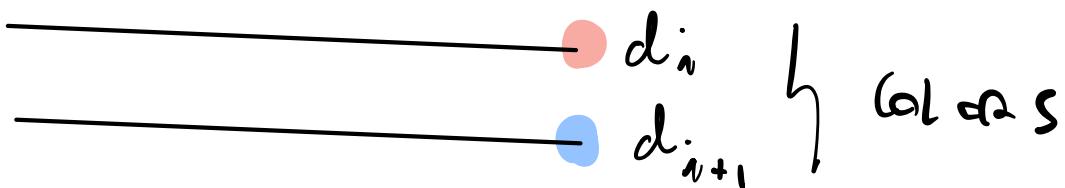
SO WHAT WE NEED TO PROVE IS

$$\text{IF } \Delta_i^+ w \geq w$$

$$\mathcal{Z}(S_{\Delta_i^+, z_i^+ w}) = \partial_i^+ \mathcal{Z}(S_{\lambda, w}).$$

WE CAN PROVE THIS USING YBE.

THE CONDITION $\Delta_i^+ w \geq w$ AMOUNTS $d_i \geq d_{i+1}$.



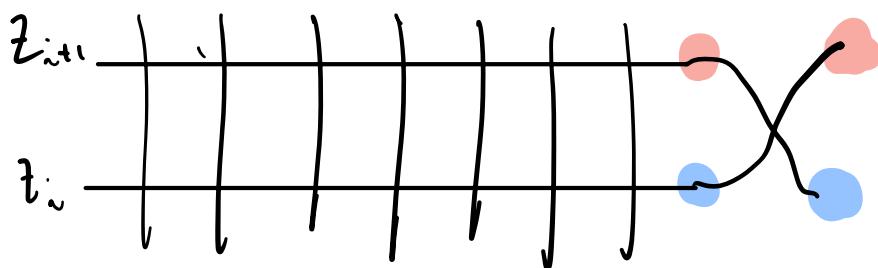
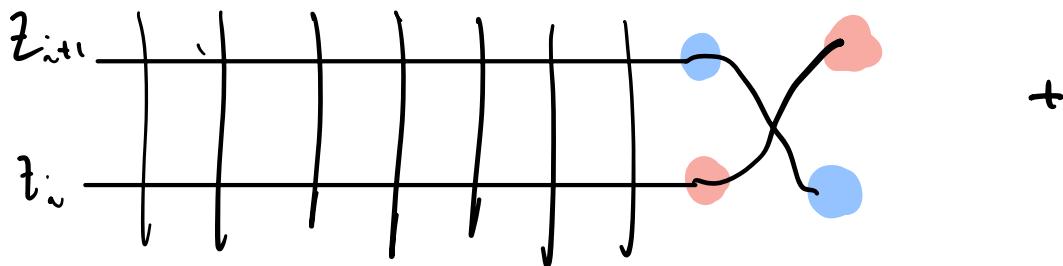
$$\alpha(z_i^+, z_{i+1}^+) :$$

$$\beta \left(\begin{array}{c} t \\ z \\ w \end{array} \right)$$

Open R-matrix			
$w \oplus_{z,w} \oplus z$	$w \circ_{z,w} \circ z$	$w \oplus_{z,w} \oplus z$	$w \circ_{z,w} \circ z$
$z \oplus \oplus w$	$z \circ \circ w$	$z \circ \circ w$	$z \circ \circ w$
w	z	z	$z \text{ if } c < d$ $w \text{ if } c > d$
$w \circ_{z,w} \circ z$	$w \circ_{z,w} \oplus z$	$w \oplus_{z,w} \circ z$	$w \circ_{z,w} \circ z$
$z \oplus \oplus w$	$z \oplus \circ w$	$z \circ \oplus w$	$z \circ \circ w$
w	0	$z - w$	$z - w \text{ if } c > d$ $0 \text{ if } c < d$

ATTACHING R-MATRIX GIVES

$$a(z_i, z_{i+1}) \mathcal{Z}(S_\lambda(z)) =$$



$$B(z_i, z_{i+1}) = \beta_{z_i, z_{i+1}} \left(\begin{array}{cc} \text{blue} & \text{red} \\ \text{red} & \text{blue} \end{array} \right) \quad R > B$$

$$C(z_i, z_{i+1}) = \beta_{z_i, z_{i+1}} \left(\begin{array}{cc} \text{red} & \text{red} \\ \text{blue} & \text{blue} \end{array} \right)$$

$$B(z_i, z_{i+1}) \mathcal{Z}(S_{\lambda, \Delta_{i,0}}(1, z)) + C(z_i, z_{i+1}) \mathcal{Z}(S_{\lambda, \Delta_{i,0}}(0, z))$$

$$a(z_i, z_{i+1}) \mathcal{Z}(S_{\lambda, w}(z)) =$$

$$b(z_i, z_{i+1}) \mathcal{Z}(S_{\lambda, \Delta_{i+1}}(z)) +$$

$$c(z_i, z_{i+1}) \mathcal{Z}(S_{\lambda, w}(z)) .$$

REPLACE $z \rightarrow \lambda \cdot z$ $z_i \leftrightarrow z_{i+1}$

$$\mathcal{Z}(S_{\lambda, \Delta_{i+1}}(z)) = \frac{c(z_{i+1}, z_i) \mathcal{Z}(S_{\lambda, w}(z)) - a(z_{i+1}, z_i) \mathcal{Z}(S_{\lambda, w}(\lambda \cdot z))}{-b(z_{i+1}, z_i)}$$

$$\mathfrak{T}_i^{\pm} f(\mathbf{z}) = \frac{c^{\pm}(z_{i+1}, z_i) f(\mathbf{z}) - a(z_{i+1}, z_i) f(s_i \mathbf{z})}{-b^{\pm}(z_{i+1}, z_i)} \quad (6.18)$$

Open R-matrix			
$w \begin{smallmatrix} \oplus \\ z, w \end{smallmatrix} z$	$w \begin{smallmatrix} c \\ z, w \end{smallmatrix} c z$	$w \begin{smallmatrix} \oplus \\ z, w \end{smallmatrix} z$	$w \begin{smallmatrix} d \\ z, w \end{smallmatrix} d z$
$z \begin{smallmatrix} \oplus \\ z, w \end{smallmatrix} w$	$z \begin{smallmatrix} c \\ z, w \end{smallmatrix} c w$	$z \begin{smallmatrix} c \\ z, w \end{smallmatrix} c w$	$z \begin{smallmatrix} c \\ z, w \end{smallmatrix} c w$
w	z	z	$z \quad \text{if } c < d$ $w \quad \text{if } c > d$
$w \begin{smallmatrix} c \\ z, w \end{smallmatrix} c z$	$w \begin{smallmatrix} c \\ z, w \end{smallmatrix} \oplus z$	$w \begin{smallmatrix} \oplus \\ z, w \end{smallmatrix} c z$	$w \begin{smallmatrix} d \\ z, w \end{smallmatrix} c z$
$z \begin{smallmatrix} \oplus \\ z, w \end{smallmatrix} w$	$z \begin{smallmatrix} \oplus \\ z, w \end{smallmatrix} c w$	$z \begin{smallmatrix} c \\ z, w \end{smallmatrix} \oplus w$	$z \begin{smallmatrix} c \\ z, w \end{smallmatrix} d w$
w	0	$z - w$	$z - w \quad \text{if } c > d$ $0 \quad \text{if } c < d$

$$C(z_i, z_{i+1}) = z_i \quad z_{i+1} = a(z_i, z_{i+1})$$

$$B(z_i, z_{i+1}) = z_i - z_{i+1}$$

$$Z(S_{\lambda, \alpha; \omega}(t)) = \frac{z_{i+1} - z_i \Delta_i}{z_i - z_{i+1}} Z(S_{\lambda, \omega}(t))$$

$$Z^\alpha := z_i / z_{i+1}$$

$$\frac{1 - Z^\alpha \Delta_i}{Z^\alpha - 1} = Z^\rho \partial_i^\alpha Z^{-\rho}.$$

$$\partial_i^\alpha = (Z^\alpha - 1)^{-1} (1 - \Delta_i)$$

$$\Delta_i \rho = \rho - \alpha_i$$

$$\Delta_i Z^{-\rho} = Z^{-\rho} \cdot Z^\alpha \Delta_i$$

$$Z^\rho \Delta_i Z^{-\rho} = Z^\alpha \Delta_i$$

APPLYING TO \mathcal{Z}^P . $\partial_w \mathcal{Z}^P = \mathcal{Z}(S_{\lambda, w}(z))$

GETS $\mathcal{Z}^P \mathcal{Z}(S_{\lambda, \lambda_w})$.

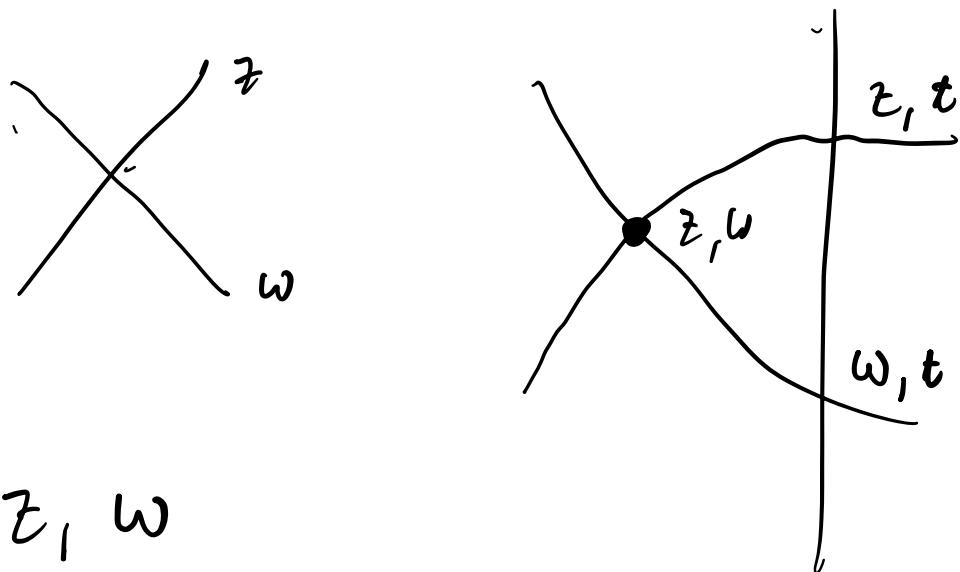
THIS IS A MODEL FOR MANY SITUATIONS
WHERE

PARAMETERIZED
YBE \rightsquigarrow DENAZURE
OPERATOR
RECURSION
FORMULA.

KEY POINT: $\beta \left(\begin{array}{c} \text{blue} \\ \times \\ \text{red} \end{array} \right) = \text{CONSTANT} \times \mathcal{Z}_i - \mathcal{Z}_{i+1}$

BECAUSE THIS GOES UP IN THE DENOMINATOR.

KIRILLOV
BRUBAKER-DAISTER } MANY EXAMPLES
PIPE DREAMS ...

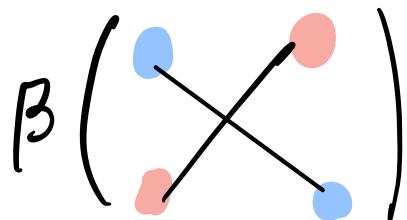


$$R_{z,w} : V \otimes V \rightarrow V \otimes V$$

$$(R_{z,w} \otimes I)(I \otimes R_{z,t})(R_{w,t} \otimes I) = (I \otimes R_{w,t})(\dots)(\dots)$$

General SA APC.

IF $z = w$, $R_{z,w}$ SHOULD BE A
MULTIPLE OF $I_{V \otimes V}$.



IS AN OFF DIAGONAL TERM!

$$\left\langle \begin{array}{c} \text{blue dot} \\ w \\ \text{red dot} \\ z \end{array} \mid R \mid \begin{array}{c} \text{red dot} \\ z \\ \text{blue dot} \\ w \end{array} \right\rangle$$

SHOULD VANISH IF $z = w$.

THIS MEANS THIS GENERAL CALCULATION
ALWAYS PRODUCES A DIVIDED DIFFERENCE
OPERATOR.