Math 210C Homework 6

- Section 6 (p.30) # 4,
- Section 10 (p.54) # 9,12,
- Section 13 (p.72) # 9.

Note: I am using some slightly different notations from Humphreys in the lectures and in the statements of the homework problems. As Humphreys explains in a note at the end of the book, notations standardized to a large extent after the book was written. Here are some differences between my notation and his.

I am using the more standard notation s_α for the "simple reflection" r_α when $\alpha \in \Delta$.

I am using the notation w_0 for the "long element" of the Weyl group, which is now very standard.

I am denoting by ρ the Weyl vector

$$\rho = \frac{1}{2} \sum_{\alpha \in \Phi^+} \alpha.$$

This is denoted δ by Humphreys, but the notation ρ is now very standard.

Section 6 (p. 30) #4. Use Weyl's theorem to give another proof that if L is semisimple, then $\mathrm{ad}(L) = \mathrm{Der}(L)$. [**Hints:** If $\delta \in \mathrm{Der}(L)$, make the direct sum $F + L$ into an L-module by the rule

$$x \cdot (a, y) = (0, a\delta(x) + [x, y]).$$

Then consider a complement to the submodule L.]

Section 10 (p. 54) #9. Prove that there is a unique element $w_0 \in W$ sending Φ^+ to Φ^-. Prove that any reduced expression for w_0 must involve all simple reflections s_α ($\alpha \in \Delta$).

Section 10 (p. 54) #12. Let $\lambda \in \mathfrak{C}(\Delta)$. If $w\lambda = \lambda$ for some $w \in W$, then $w = 1$.

Remark: The following stronger statement is true: If $\lambda \in \mathfrak{C}(\Delta)$ and $w \in W$ such that $w\lambda \in \mathfrak{C}(\Delta)$, then $w = 1$. If you prove this stronger statement it may be helpful in the following problem as well.

Section 13 (p. 72) #9. Let $\lambda \in \Lambda^+$. Prove that for $\sigma \in W$ the weight $\sigma(\lambda + \rho) - \rho$ is dominant only if $\sigma = 1$.