Math 210C Homework 5

- Section 9 (p.45) # 2,6,
- Section 10 (p.54) # 2,6.

Section 9, Problem 2. Prove that $\Phi^\vee$ is a root system in E, whose Weyl group is naturally isomorphic to $\mathcal{W}$; show also that

$$\frac{2(\beta^\vee, \alpha^\vee)}{(\alpha^\vee, \alpha^\vee)} = \frac{2(\alpha, \beta)}{(\beta, \beta)}$$

and draw a picture of $\Phi^\vee$ in the cases $\mathtt{A}_2$, $\mathtt{B}_2$, $\mathtt{G}_2$.

Section 9, Problem 6. Prove that $\mathcal{W}$ is a normal subgroup of $\mathrm{Aut}(\Phi)$, the group of all linear isomorphisms of Φ onto itself.

Section 10, Problem 2. If Δ is a base of Φ, and $\alpha, \beta \in \Delta$ ($\alpha \neq \beta$), prove that the set $(\mathbb{Z}\alpha + \mathbb{Z}\beta) \cap \Phi$ is a root system of rank 2 in the subspace E spanned by α, β (see Exercise 9.7).

Section 10, Problem 6. Define a function $\mathrm{sn} : \mathcal{W} \longrightarrow \{\pm 1\}$ by $\mathrm{sn}(\sigma) = (-1)^{\ell(\sigma)}$. Prove that sn is a homomorphism.