

# Math 210C Homework 3

- Section 4 (page 20) #7
- Section 5 (page 24) #1,5
- Section 6 (page 30) #6,7

**Section 4 #7** Prove the converse to Theorem 4.3. That is, if  $L \subseteq gl(V)$  is solvable prove that  $\text{tr}(xy) = 0$  for all  $x \in [L, L]$  and  $y \in L$ .

**Section 5 #1** Prove that if  $L$  is nilpotent, the Killing form of  $L$  is identically zero.

**Section 5 #5** Let  $L = \mathfrak{sl}(2, F)$ . Compute the basis of  $L$  dual to the standard basis, relative to the Killing form.

**Section 6 #6** Let  $L$  be a simple Lie algebra. Let  $\beta(x, y)$  and  $\gamma(x, y)$  be two symmetric associative bilinear forms on  $L$ . If  $\beta, \gamma$  are nondegenerate, prove that  $\beta$  and  $\gamma$  are proportional. [Hint: Use Schur's Lemma.]

**Section 6 #7** It will be seen later that  $\mathfrak{sl}(n, F)$  is actually *simple*. Assuming this and using Exercise 6, prove that the Killing form  $\kappa$  on  $\mathfrak{sl}(n, F)$  is related to the ordinary trace bilinear form by  $\kappa(x, y) = 2n \text{tr}(xy)$ .