Math 210C Homework 2

- Humphreys Section 2 (page 9) #1,7
- Section 3 (page 14) #1,2,6
- Section 4 (page 20) #1,5

Problem 1: Humphreys Section 2 #1. Prove that the set of all inner derivations $\mathrm{ad}(x)$, $x \in L$ is an ideal of $\mathrm{Der}(L)$.

Note: Humphreys uses this notation: $\mathfrak{t}(n, F)$ is the subalgebra of $\mathfrak{gl}(n, F)$ consisting of upper triangular matrices, $\mathfrak{n}(n, F)$ =strictly upper triangular matrices, and $\mathfrak{d}(n, F)$ =diagonal matrices. In the lectures I am using the following notations, which are more widely used.

Humphreys	Us	Common name
$\mathfrak{t}(n, F)$	$\mathfrak{b}$	"Borel subalgebra"
$\mathfrak{n}(n, F)$	$\mathfrak{n}$ or $\mathfrak{n}^+$	
$\mathfrak{d}(n, F)$	$\mathfrak{h}$	"Cartan subalgebra"

Problem 2: Section 2 #7. Prove that $\mathfrak{t}(n, F)$ and $\delta(n, F)$ are self-normalizing subalgebras of $\mathfrak{gl}(n, F)$, whereas $\mathfrak{n}(n, F)$ has normalizer $\mathfrak{t}(n, F)$.

Problem 3: Section 3 #1. Let I be an ideal of L. Then each member of the derived series or descending central series of I is an ideal of L.

Problem 4: Section 3 #2. Prove that L is solvable if and only if there exists a chain of subalgebras $L = L_0 \supset L_1 \supset L_2 \supset \cdots \supset L_k = 0$ such that L_{i+1} is an ideal of L_i and such that each quotient L_i/L_{i+1} is abelian.

Problem 5: Section 3 #6. Prove that a sum of two nilpotent ideals of a Lie algebra L is again a nilpotent ideal. Therefore L possesses a unique maximal nilpotent ideal. (Humphreys asks you to determine this for particular algebras but you may skip this part.)

Problem 6: Section 4 #1. Let $L = \mathfrak{sl}(V)$. Use Lie's Theorem to prove that $L = Z(L)$; conclude that L is semisimple. (See book for hint.)

Problem 7: Section 4 #5. If $x, y \in \mathrm{End}(V)$ commute, prove that $(x + y)_s = x_s + y_s$ and $(x+y)_n = x_n + y_n$. Show by example that this can fail if x, y fail to commute. (See book for hint.)