Correction to the proof of JDT.

We show that if $x_1, \cdots, x_n \in M$ then there exists $r \in R$ such that

$$\phi(x_i) = r x_i.$$

If this is known, then we can take $x_1, \cdots, x_n$ to be a basis of M and JDT is proved.

Dihedral group

$$\langle t, s \,|\, t^4 = s^2 = 1, s t s^{-1} = t^{-1} \rangle$$

	1	t	t^2	s	st
	1	(1234)	(13)(24)	(13)	(12)(34)
χ_1	1	1	1	1	1
χ_2	1	1	1	-1	-1
χ_3	1	-1	1	1	-1
χ_4	1	-1	1	-1	1
χ_5	2	0	-2	0	0
χ_{reg}	8	0	0	0	0
$\overset{\circ}{\chi}_{\text{perm}}$	3	-1	-1	1	-1

$$\chi_5 = \overset{\circ}{\chi}_{\text{perm}} - \chi_3$$

Linear characters are characters of G/G' and $G' = Z(G) = \langle t^2 \rangle$

$$G / \langle t^2 \rangle = \langle \bar{t}, \bar{s} \,|\, \bar{t}^2 = \bar{s}^2 = 1, \bar{s}\bar{t}\bar{s}^{-1} = \bar{t} \rangle \cong Z_2 \times Z_2$$

$$1 + 1 + 1 + 1 + d_5^2 = d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 = 8$$

$$d_5 = 2$$

$$\chi_{\text{reg}} = \chi_1 + \chi_2 + \chi_3 + \chi_4 + 2\chi_5$$

$$\chi_5 = \frac{1}{2}(\chi_{\text{reg}} - \chi_1 - \chi_2 - \chi_3 - \chi_4)$$

The regular representation π_{reg} is the permutation representation of G acting on itself.

$$\chi_{\text{reg}}(g) = \begin{cases} |G| & \text{if } g = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$gx = x \quad \Rightarrow \quad g = 1.$$

$$\chi_{\text{reg}} = \sum d_i \chi_i$$

$$\text{Mat}_2(\mathbb{C}) = M_1 \oplus M_2$$

$$M_1 = \begin{pmatrix} * & 0 \\ * & 0 \end{pmatrix} \cong \mathbb{C}^2 \quad \text{as Mat}_2(\mathbb{C})\text{-module}$$

$$M_2 = \begin{pmatrix} 0 & * \\ 0 & * \end{pmatrix}$$

Characters of S_4

	1	8	3	6	6
	1	(123)	(12)(34)	(12)	(1234)
χ_1	1	1	1	1	1
χ_2	1	1	1	-1	-1
χ_3	2	-1	2	0	0
χ_4	3	0	-1	1	-1
χ_5	3	0	-1	-1	1

$$S_4 / S_4' = S_4 / A_4 \cong Z_2$$

$$(12)(123)(12)(132) = (123)$$

$$V = \{1, (12)(34), (13)(23), (14)(23)\}$$

$$S_4 / V \cong 24/4 = 6$$

$$S_4 / V \cong S_3$$

Note S_4 acts on the set

$$X = \{(12)(34), (13)(23), (14)(23)\}$$

by conjugation. This permatutation representation gives a homomorphism $S_4 \longrightarrow S_3$ with kernel V.

	1	(132)	(12)
ψ_1	1	1	1
ψ_2	1	1	-1
ψ_3	2	-1	0

$$1^2 + 1^2 + 2^2 + d_3^2 + d_4^2 = \sum d_i^2 = 24$$

$$d_3^2 + d_4^2 = 18$$

$$d_3 = d_4 = 3$$

$$\langle \chi_4, \chi_4 \rangle = \frac{1}{24}(3^2 + 8 \times 0 + 3 \times (-1)^2 + 6 \times 1^2 + 6 \times (-1)^2) = 1$$