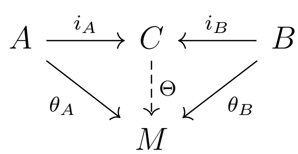
Homework 3

Do the following problems in Dummit and Foote.

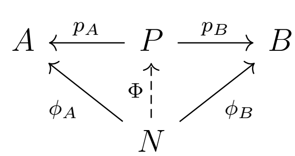
- Section 10.3 problem 20
- Section 18.1 problems 1–4

Let $\mathcal{C}$ be a category, and A, B objects. A *coproduct* of A and B consists of an object C together with morphisms $i_A : A \to C$ and $i_B : B \to C$ such that the following property is satisfied.

Universal property of the coproduct: Given any object M with morphisms $\theta_A : A \to M$ and $\theta_B : B \to M$ there is a unique morphism $\Theta : C \to M$ such that $\theta_A = \Theta \circ i_A$ and $\theta_B = \Theta \circ i_B$.



A *product* of A and B consists of an object P together with morphisms $p_A : P \to A$ and $p_B : P \to B$ such that given any object N with morphisms $\phi_A : N \to A$ and $\phi_B : N \to B$ there is a unique morphism $\Phi : N \to P$ such that $\phi_A = p_A \circ \Phi$ and $\phi_B = p_B \circ \Phi$.



Problem A. Explain briefly why if the coproduct or product exists in a category (for fixed A and B) it is unique up to isomorphism.

Problem B. In the category of R-modules, prove that the Cartesian product $A \times B$ is both a product and a coproduct.

Problem C. What are the product and coproduct in the category of sets?