MATH 122: HOMEWORK 6

- Section 18.3 # 11*,12,13
- Section 19.1 # 7*,14,15

(*) Problems 18.3 # 11 and 19.1 # 7 were previously assigned. If you are confident in your previous solutions you do not need to do these.

Section 18.2 #11. Let χ be an irreducible character of G. Prove that for every element z in the center of G we have $\chi(z) = \varepsilon\chi(1)$ where ε is some root of unity in $\mathbb{C}$. [**Hint:** Use Schur's Lemma]

Section 18.2 #12. Let ψ be the character of some representation φ of G. Prove that for $g \in G$ the following hole.
(a) If $\psi(g) = \psi(1)$ then $g \in \mathrm{Ker}(\varphi)$;
(b) If $|\psi(g)| = \psi(1)$ and φ is faithful then $g \in Z(G)$ (where $|\psi(g)|$ is the complex absolute value of $\psi(g)$. [**Hint:** Use the method of proof of Proposition 14.]

Section 18.2 #13. Let $\varphi : G \longrightarrow \mathrm{GL}(V)$ be a representation and let $\chi : G \longrightarrow \mathbb{C}^\times$ be a degree 1 representation. Prove that $\chi\varphi : G \longrightarrow \mathrm{GL}(V)$ defined by $(\chi\varphi)(g) = \chi(g)\varphi(g)$ is a representation (note that multiplication of the linear transformation $\varphi(f)$ by the complex number $\chi(g)$ is well-defined.) Show that $\chi\varphi$ is irreducible if and only if φ is irreducible. Show that if ψ is the character afforded by φ then $\chi\psi$ is the character afforded by $\chi\varphi$. Deduce that the product of any irreducible character with a character of degree 1 is also an irreducible character.

Section 19.1 #7. Show that S_6 has an irreducible character of degree 5.

Section 19.1 #14. Let n be an integer with $n \geqslant 3$. Show that every irreducible character of D_{2n} has degree 1 or 2 and find the number of irreducible characters of each degree. (The conjugacy classes of D_{2n} were found in Exercises 31 and 32 of Section 4.3 and its commutator subgroup was computed in Section 5.4.)

Section 19.1 #15. Prove that the character table is an invertible matrix.