MATH 122: HOMEWORK 5

- Section 18.3 # 6,7
- Section 19.1 #2,3,7,8,9

Section 18.3 #6. Let $\varphi : G \longrightarrow \mathrm{GL}(V)$ be a representation with character ψ. Let W be the subspace

$$W = \{v \in V | \varphi(g)v = v \text{ for all } g \in G\}$$

fixed pointwise by all elements of G. Prove that $\dim(W) = (\psi, \chi_1)$ where χ_1 is the principal character of G.

Section 18.3 #7. Let Assume that V is a $\mathbb{C}[G]$-module on which G acts by permuting the basis $\mathcal{B} = \{e_1, \cdots, e_n\}$. Write $\mathcal{B}$ as a disjoint union of orbits $\mathcal{B}_1, \cdots, \mathcal{B}_n$ of G on $\mathcal{B}$.

(a) Prove that V decomposes as a $\mathbb{C}[G]$-module as $V_1 \oplus \cdots \oplus V_t$ where V_i is the span of $\mathcal{B}_i$.

(b) Prove that if v_i is the sum of the vectors in $\mathcal{B}_i$, then v_i is the unique $\mathbb{C}[G]$-submodule of V_i affording the trivial representation (in other words, any vector in V_i that is fixed under the action of G is a multiple of v_i. [Use the fact that G is transitive on $\mathcal{B}_i$. See also Exercise 8 in Section 1.1.

(c) Let $W = \{v \in V | \varphi(g)v = v \text{ for all } g \in G\}$. Prove that $\dim(W) = t$ is the number of orbits of G on $\mathcal{B}$.

Section 19.1 #2. Compute the degrees of the irreducible characters of D_{16}.

Section 19.1 #3. Compute the degrees of the irreducible characters of A_5. Deduce that the degree 6 irreducible representation is not irreducible when restricted to A_5. [The conjugacy classes of A_5 are worked out in Section 4.3.

Section 19.1 #7. Show that S_6 has an irreducible character of degree 5.

Section 19.1 #8. Calculcate the character table of D_{10}. (This table contains nonreal entries.)

Section 19.1 #9. Calculcate the character table of D_{12}.