MATH 122: HOMEWORK 4

- Section 18.3 # 1,2a,4,5,11,20

Section 18.3 #1. Prove that $\mathrm{tr}(AB) = \mathrm{tr}(BA)$ for $n \times n$ matrices A and B with entries in any commutative ring.

Section 18.3 #2a. Let φ be the degree 2 representation of D_{10} desribed in Example 6 in the second set of examples in Section 1 (here $n = 5$) and show that $\|\varphi\|^2 = 1$. Deduce that φ is irreducible.

Section 18.3 #4. Prove that if N is any irreducible $\mathbb{C}G$-module and $M = N \oplus N$ then M has infinitely many direct sum decompositions into two copies of N.

Section 18.4 #5. Prove that a class function is a character if and only if it is a positive integral linear combination of irreducible characters.

Section 18.4 #11. Let χ be an irreducible character of G. Prove that for every element z in the center of G we have $\chi(z) = \varepsilon\chi(1)$ where ε is some root of 1 in $\mathbb{C}$. [**Hint:** Use Schur's Lemma.]

Section 18.4 #20. Prove that elements x and y of G are conjugate in G if and only if $\chi(x) = \chi(y)$ for all irreducible characters χ of G.