MATH 122: HOMEWORK 3

- Section 18.1 # 3,11,15,16,20
- Section 18.2 # 3

Let G be a finite group. The *commutator subgroup* G', also called the *derived subgroup* is the subgroup generated by all commutators $[x, y] = xyx^{-1}y^{-1}$. The word "generated by" is important since the product of commutators may not be a commutator. See Dummit and Foote Proposition 7 on page 169.

Section 18.1 #3. Prove that the degree 1 representations of G are in bijection with the degree 1 representations of G/G'.

Section 18.1 #11. Let $\varphi : S_n \to \mathrm{GL}_n(F)$ be the matrix representation given by the permutation module described in Example 3 in the second set of examples, where the matrices are computed with respect to the basis $e_1, \cdots, e_n$. Prove that $\det(\varphi(\sigma)) = \varepsilon(\sigma)$ for all $\sigma \in S_n$, where $\varepsilon(\sigma)$ is the sign of the permutation σ. [**Hint:** Check this on transpositions.]

Section 18.1 #15. Exhibit all 1-dimensional complex representations of a finite cyclic group; make sure to decide which are inequivalent.

Section 18.1 #16. Exhibit all 1-dimensional complex representations of a finite abelian group; deduce that the number of inequivalent degree 1 complex representations of a finite abelian group equals the order of the group. [First decompose the abelian group into a direct product of cyclic groups, then use the preceding exercise.]

Section 18.1 #20. Prove that the number of degree 1 complex representations of any finite group equals $[G : G']$, where G' is the commutator subgroup of G. [**Hint:** use Exercises 3 and 16.]

Section 18.2 #3. Prove that (4) implies (3) in Wedderburn's Theorem. [**Hint:** Let N be a nonzero R-module. First show that N contains simple submodules by considering a cyclic submodule. Then use Zorn's Lemma applied to the set of direct sums of simple submodules (appropriately ordered) to show that N contains a maximal completely reducible submodule M. If $M \neq N$ let M_1 be teh complete preimage in N of a simple module in N/M and contradict the maximality of M.]