Math 121 Homework 5 Solutions

- Section 14.1 # 4,10
- Section 14.2 # 1,5,6,13.

Problem 4, Section 14.1. Prove that $\mathbb{Q}\left(\sqrt{2}\right)$ and $\mathbb{Q}\left(\sqrt{3}\right)$ are not isomorphic.

Problem 10, Section 14.1. Let K be an extension of the field F. Let $\varphi : K \longrightarrow K'$ be an isomorphism of K with a field K' which maps F to the subfield F' of K'. Prove that the map $\sigma \mapsto \varphi\sigma\varphi^{-1}$ defines a group isomorphism $\mathrm{Aut}(K/F) \longrightarrow \mathrm{Aut}(K'/F')$.

Problem 1, Section 14.2. Determine the minimal polynomial over $\mathbb{Q}$ for the element $\sqrt{2}+\sqrt{5}$.

Section 14.2, Problem 5. Prove that the Galois group of $x^p - 2$ over $\mathbb{Q}$ for p a prime is isomorphic to the group of matrices $\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}$ for $a, b \in \mathbb{F}_p$, $a \neq 0$.

Section 14.2, Problem 6. Let $K = \mathbb{Q}\left(\sqrt[8]{2}, i\right)$ and let $F_1 = \mathbb{Q}(i)$, $F_2 = \mathbb{Q}\left(\sqrt{2}\right)$, $F_3 = \mathbb{Q}\left(\sqrt{-2}\right)$. Prove that $\mathrm{Gal}(K/F_1) \cong Z_8$, $\mathrm{Gal}(K/F_2) \cong D_8$ and $\mathrm{Gal}(K/F_3) \cong Q_8$.

Section 14.2, Problem 13. Show that $\mathbb{Q}\left(\sqrt{2+\sqrt{2}}\right)$ is a cyclic quartic field, that is, it is Galois over $\mathbb{Q}$ of degree 4 with cyclic Galois group.