Math 121 Homework 4

- Section 9.4 # 2a, 2c, 7;
- Section 13.6 # 3, 7;
- Section 14.1 # 2, 3.

Problem 2, Section 9.4. Prove that the following polynomials are irreducible in $\mathbb{Z}[x]$. (a) $x^4 - 4x^3 + 6$;

(c) $x^4 + 4x^3 + 6x^2 + 2x + 1$ (Substitute $x - 1$ for x.)

Remark: By Gauss' Lemma (Proposition 5 on page 303), a monic polynomial that is irreducible in $\mathbb{Z}[x]$ is irreducible in $\mathbb{Q}[x]$. So these polynomials are irreducible in $\mathbb{Q}[x]$.

Problem 7, Section 9.4. Prove that $\mathbb{R}[x]/(x^2 + 1)$ is a field which is isomorphic to the field of complex numbers.

Problem 3, Section 13.6. Prove that if a field F contains the n-th roots of unity for n odd, then it contains the $2n$-th roots of unity.

Problem 7, Section 13.6. Use the Möbius Inversion Formula indicated in Section 14.3 to prove

$$\Phi_m(x) = \prod_{d|n} (x^d - 1)^{\mu(m/d)}.$$

Problem 2, Section 14.1. Let $\tau : \mathbb{C} \longrightarrow \mathbb{C}$ be the complex conjugation, defined by $\tau(a + bi) = a - bi$. Prove that τ is an automorphism of $\mathbb{C}$.

Problem 3, Section 14.1. Determine the fixed field of complex conjugation on $\mathbb{C}$.